Part 1 - Pre-Algebra Summary

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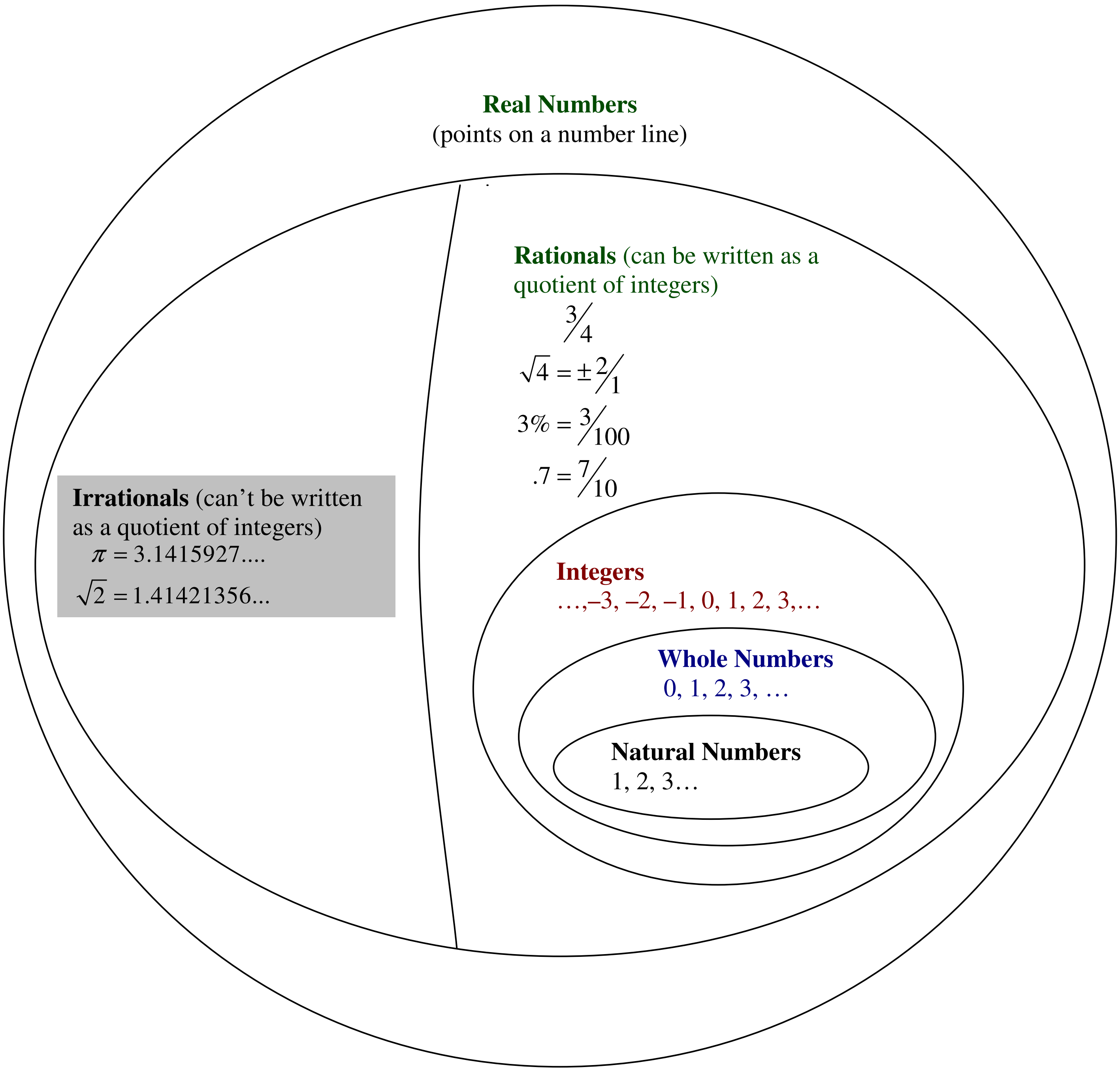
1. Numbers

1.1. Names for Numbers



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1.2. Place Values

Hundred-Millions	Ten-Millions	Millions	Hundred-Thousands	Ten-Thousands	Thousands	Hundreds	Tens	Ones	"AND"	Tenths	Hundredths	Thousandths	Ten-Thousandths
9	8	7	6	5	4	3	2	1	.	1	2	3	4
100,000,000	10.000.000	1,000,000	100,000	10,000	1000	100	10	1	Decimal Point	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$	$\frac{1}{10,000}$

- **Place values** – go up by powers of 10. 234 can be expressed as $(2 \bullet 100) + (3 \bullet 10) + (4 \bullet 1)$
- **Commas** – In numbers with more than 4 digits, commas separate off each group of 3 digits, starting from the right. These groups are read off together.
- **Reading numbers** – 123,406,009.023 = "One hundred twenty-three ***million***, four hundred six ***thousand***, nine and twenty-three ***thousandths***"

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1.3. Inequalities

< **Less than** > **Greater than** ≤ **Less than or equal to** ≥ **Greater than or equal to** = **Equal** ≠ **Not equal to**	▪ Mouth "eats" larger number  ▪ Represent numbers on a number line. The number to the right is always greater –7 –6 0 6 7 ▪ Convert numbers so everything is "the same" – e.g. all decimals, all fractions with common denominators, etc.	▹ 7 > 6 ▹ 6 < 7 ▹ –7 <– 6 ▹ –6 > –7 ▹ –5/3 ? –.01 –.67 < –.01 ▹ –4/2 ? –10/4 –8/4 > –10/4

1.4. Rounding

1. Locate the digit to the right of the given place value 2. If the digit is ≥ 5, add 1 to the digit in the given place value 3. If the digit is < 5, the digit in the given place value remains 4. Zero/drop remaining digits	Round to the nearest hundreds: ▹ 1220 = 1200 ▹ 1255.12 = 1300 Round to the nearest hundredths: ▹ 25.1254 = 25.13 ▹ 25.1230 = 25.12

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1.5. Divisibility Tests

2	If last digit is 0,2,4,6, or 8	22, 30, 50, 68, 1024
3	If sum of digits is divisible by 3	123 is divisible by 3 since 1 + 2 + 3 = 6 (and 6 is divisible by 3)
4	If number created by the last 2 digits is divisible by 4	864 is divisible by 4 since 64 is divisible by 4
5	If last digit is 0 or 5	5, 10, 15, 20, 25, 30, 35, 2335
6	If divisible by 2 & 3	522 is divisible by 6 since it is divisible by 2 & 3
9	If sum of digits is divisible by 9	621 is divisible by 9 since 6 + 2 + 1 = 9 (and 9 is divisible by 9)
10	If last digit is 0	10, 20, 30, 40, 50, 5550

1.6. Properties of Real Numbers

	For Addition	**For Subtraction**	**For Multiplication**	**For Division**
Commutative	$a + b = b + a$	$a - b \neq b - a$	$ab = ba$	$a/b \neq b/a$
Associative	$(a+b)+c = a+(b+c)$	$(a-b)-c \neq a-(b-c)$	$(ab)c = a(bc)$	$(a \div b) \div c \neq a \div (b \div c)$
Identity	$0+a = a$ & $a+0 = a$	$a - 0 = a$	$a \bullet 1=a$ & $1 \bullet a=a$	$a \div 1 = a$
Inverse	$a + (-a) = 0$ & $(-a) + a = 0$	$a - a = 0$	$1/a \bullet a = 1$ & $a \bullet 1/a = 1$ if $a \neq 0$	$a \div a = 1$ if $a \neq 0$
Distributive Property	$a(b + c) = ab + ac$	$a(b - c) = ab - ac$	$-a(b + c) = -ab - ac$	$-a(b - c) = -ab + ac$

1.7. Properties of Equality

Addition Property of Equality	If $a = b$ then $a + c = b + c$
Multiplication Property of Equality	If $a = b$ then $ac = bc$
Multiplication Property of 0	$0 \bullet a = 0$ and $a \bullet 0 = 0$

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1.8. Order of Operations

① **SIMPLIFY ENCLOSURE SYMBOLS:** Absolute value $\| \|$, parentheses ( ), or brackets [ ] ❖ If multiple enclosure symbols, do innermost 1st ❖ If fraction, pretend it has ( ) around its numerator and ( ) around its denominator	▷ $\dfrac{3\|-2\|+12}{2[-3(2+1)]}$ $=\dfrac{3\times 2+12}{2(-3(3))}$ $=\dfrac{6+12}{2(-9)}$ $=-\dfrac{\cancel{18}}{\cancel{18}}$ $=-1$
② **CALCULATE EXPONENTS** (Left to Right)	▷ Evaluate x^2 for $x=-5$ (means the base is -5, not 5! It's very helpful to put the value of the variable in parentheses) $x^2=(-5)^2=(-5)(-5)=25$ ▷ $-(-5)^2=-(-5)(-5)=-(25)=-25$ ▷ $\|5\|^2=5^2=25$ ▷ $\|-5\|^2=5^2=25$
③ **PERFORM MULTIPLICATION & DIVISION** (Left to Right)	▷ $5-\underline{2\bullet 10}+\underline{30\div 10}\bullet 2$ $=5-20+3\bullet 2$
④ **PERFORM ADDITION & SUBTRACTION** (Left to Right)	$=5-20+6$ $=5+(-20)+6$ $=-15+6$ $=-9$
⑤ **SIMPLIFY FRACTIONS**	▷ $\dfrac{5}{1}=5$ ▷ $\dfrac{24}{36}=\dfrac{\cancel{6}\bullet\cancel{2}\bullet 2}{\cancel{6}\bullet 3\bullet\cancel{2}}=\dfrac{2}{3}$ ▷ $\dfrac{30}{9}=\dfrac{\cancel{3}\bullet 10}{\cancel{3}\bullet 3}=\dfrac{10}{3}$ OR $3\dfrac{1}{3}$

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2. Real Numbers

2.1. Operations		
Absolute Value $\lvert x\rvert$	❖ The DISTANCE (which is always positive) of a number from zero on the number line	▷ $\lvert 2\rvert = 2$ ▷ $-\lvert 2\rvert = -2$ ▷ $\lvert -2\rvert = 2$ ▷ $-\lvert -2\rvert = -2$ ▷ $\lvert 0\rvert = 0$ ▷ $-\lvert -2\rvert^2 = -(2\bullet 2) = -4$
Addition $+$	❖ If the signs of the numbers are the SAME, ADD absolute values ❖ If the signs of the numbers are DIFFERENT, SUBTRACT absolute values ❖ The answer has the sign of the number with the largest absolute value	▷ $3+(-2)=1$ addend addend sum ▷ $-2+3=1$ ▷ $-3+(-2)=-5$ ▷ $-3+(2)=-1$
Subtraction $-$	❖ Change subtraction to ADDITION OF THE OPPOSITE NUMBER ❖ ADD numbers AS ABOVE	▷ $3-2=3+(-2)=1$ minuend subtrahend difference ▷ $-2-(-3)=-2+3$ ▷ $-3-2=-3+(-2)$ ▷ $-3-(-2)=-3+2$
Multiplication $\bullet$	❖ MULTIPLY the numbers ❖ DETERMINE THE SIGN OF THE ANSWER ▪ If the number of negative signs is EVEN, the answer is POSITIVE ▪ If the number of negative signs is ODD, the answer is NEGATIVE	▷ $3\bullet 2 = 3\times 2 = (3)(2) = 6$ factor factor product ▷ $(-3)(-2)=6$ ▷ $3\bullet -2 = 3\bullet(-2) = (3)(-2) = -6$ ▷ $(-3)(-2)(-2) = -12$
Division $\div$ (Divisors $\neq$ zero)	❖ DIVIDE the numbers ❖ DETERMINE THE SIGN of the answer by using the MULTIPLICATION SIGN RULES AS ABOVE	▷ $(-12)\div(-2)=6$ dividend divisor quotient ▷ $(-12)/2=-6$ ▷ $2\overline{)-12} = -6$
Exponential Notation	▪ A exponent is a shorthand way to show how many times a number (the base) is multiplied by itself ▪ An exponent applies only to the base ▪ Any number to the zero power is 1	▷ $3^0 = 1$ $3^1 = 3$ $3^2 = 3\bullet 3 = 9$ ▷ $2\bullet 5^2 = 2\bullet 5\bullet 5$ ▷ $(2\bullet 5)^2 = 2\bullet 2\bullet 5\bullet 5$ ▷ $-2^2 = -2\bullet 2$ (base is 2) ▷ $(-2)^2 = -2\bullet -2$ (base is -2)
Double Negative	❖ The opposite of a negative is POSITIVE	▷ $-(-x) = -1(-1\bullet x) = x$

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3. Fractions

3.1. Definitions		
Fractions **Proper fraction** **Improper fraction**	❖ Numerator/denominator ○ Numerator < Denominator ○ Numerator > Denominator	 ▹ 2/7 ▹ –7/2
Mixed number	❖ Integer + fraction	▹ –3 ½
Factor	❖ A whole number that divides into another number	Factors of 18: 1, 2, 3, 6, 9, & 18
Prime Number	❖ A whole number > 1 whose only factors are 1 and itself	▹ ▹ 2, 3, 5, 7, 11, 13…
Composite Number	❖ A whole number > 1 that is not prime	▹ 4, 6, 8, 9, 10…
Prime Factorization	❖ A composite number written as a product of prime numbers	▹ $18 = 2 \bullet 3 \bullet 3$
Factor Tree	❖ A method for determining prime factorization of a number	18 3 6 2 3
Lowest Terms $\frac{3}{2}$ or $1\frac{1}{2}$	❖ Numerator and denominator have <u>NO COMMON FACTORS</u> other than 1. Reduce fractions by cancelling common factors. ❖ If the numerator and/or denominator has addition or subtraction, it is not in factored form. No cancelling of factors	▹ $\frac{12}{36} = \frac{\cancel{2} \bullet \cancel{6}}{\cancel{2} \bullet 3 \bullet \cancel{6}} = \frac{1}{3}$ ▹ $\frac{\cancel{4}x + 8y}{\cancel{4}}$ NO!!
Equivalent $\frac{1}{2} = \frac{5}{10}$	❖ Numbers that represent the same point on a number line ❖ Multiplying a number by any fraction equal to 1 does not change the value of the number (see *Multiplication Identity*)	▹ $\frac{2}{3} = \frac{x}{27}$ $= \left(\frac{2}{3}\right)\left(\frac{9}{9}\right) = \frac{x}{27} = \frac{18}{27}$

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3.2. Least Common Denominators

Least Common Multiple (LCM) (Smallest number that the given numbers will divide into)	① METHOD 1 (USE THE LARGEST NUMBER) 1. Start with the largest number. Do the other numbers divide into it? 2. If yes, you're done! 3. If no, double the largest number. Do the other numbers divide into it? 4. If yes, you're done! 5. If no, triple the largest number. Do the other numbers divide into it? 6. Keep going until you're done	Ex. Find the LCM of 4,6,9 9, no 18, no 27, no 36, YES ☺
	② METHOD 2 (PRIME FACTORIZATION) 1. Determine the prime factorization of each number 2. The LCM will have every prime factor that appears in each number. Each prime factor will appear the number of times as it appears in the number which has the most of that factor.	Ex. Find the LCM of 4,6,9 4 → 2 2 6 → 2 3 9 → 3 3 LCM = 2 • 2 • 3 • 3 = 36
	③ METHOD 3 (L METHOD) 1. Find a number that divides into at least two of the numbers 2. Perform the division 3. Repeat steps 1 & 2 until there are no more numbers that divide into at least two of the numbers 4. Multiple the leftmost and bottommost numbers together	Ex. Find the LCM of 4,6,9 3 | 4 6 9 2 | 4 2 3 2 1 3 LCM = 3 • 2 • 2 • 1 • 3 = 36
Least Common Denominator (LCD)	❖ The smallest positive number divisible by all the denominators. The LCD is also the LCM of the denominators.	Ex. $\frac{1}{4}, \frac{1}{6}, \frac{1}{9}$ LCD = 36

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3.3. Operations

Convert mixed numbers to improper fractions & solve as fractions
Denominator ≠ 0; Always write final answer in lowest terms

Addition/ Subtraction $+\ -$	❖ If the DENOMINATORS are the SAME: ▪ COMBINE NUMERATORS ▪ Write answer over common denominator	▷ $\frac{1}{4}-\frac{2}{4}=\frac{-1}{4}$
	❖ If the DENOMINATORS are DIFFERENT: ▪ Write an equivalent expression using the LEAST COMMON DENOMINATOR ▪ ADD/SUBTRACT (see “If the DENOMINATORS are the SAME”)	▷ $\frac{1}{4}-\frac{1}{2}$ LCD = 4 $=\frac{1}{4}-\frac{1}{2}\left(\frac{2}{2}\right)=\frac{1}{4}-\frac{2}{4}=\frac{-1}{4}$
Multiplication $\bullet$	❖ FACTOR numerators and denominators ❖ Write problem as one big fraction ❖ CANCEL common factors ❖ MULTIPLY top × top & bottom × bottom	▷ $\frac{2}{5}\bullet\frac{25}{6}=\frac{\cancel{2}\bullet\cancel{5}\bullet 5}{\cancel{5}\bullet\cancel{2}\bullet 3}=\frac{5}{3}$
Division $\div$	❖ RECIPROCATE (flip) the divisor ❖ MULTIPLY the fractions (See *Multiplication*)	▷ $\frac{2/5}{6/25}=\frac{2}{5}\div\frac{6}{25}=\frac{2}{5}\bullet\frac{25}{6}$
Converting an Improper Fraction to a Mixed Number	1. Numerator ÷ Denominator 2. Whole-number part of the quotient is the whole-number part of the mixed number. $\frac{\text{Remainder}}{\text{Divisor}}$ is the fractional part.	▷ $-\frac{7}{2}$ $=(-7)\div 2$ $=-3$ remainder 1 $=-3\frac{1}{2}$
Converting a Mixed Number to an Improper Fraction	1. If the mixed number is negative, ignore the sign in step 2 and add the sign back in step 3 2. (Denominator × whole-number part) + numerator 3. $\frac{\text{Answer to step 2}}{\text{Original denominator}}$	▷ $-3\frac{1}{2}$ $2\bullet 3+1=7$ $=-\frac{7}{2}$

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4. Decimals

4.1. Operations

Addition/ Subtraction $+ \; -$	❖ LINE UP the decimals ❖ "Pad" with 0's ❖ Perform operation as though they were whole numbers ❖ Remember, the decimal point come straight down into the answer	▹ $1.5 + .02 + .4$ $= \begin{array}{r} 1.5\mathbf{0} \\ .02 \\ +.4\mathbf{0} \\ \hline 1.92 \end{array}$
Multiplication $\bullet$	❖ Multiply the decimals as though they were whole numbers ❖ Take the results and position the decimal point so the number of decimal places is equal to the SUM OF THE NUMBER OF DECIMAL PLACES IN THE ORIGINAL PROBLEM	▹ $1.4 \bullet 1.5$ $= 14 \bullet 15$ $= 210.$ $= 2.1$
Division $\div$	❖ If the DIVISOR contains a DECIMAL POINT ▪ MOVE THE DECIMAL POINT TO THE RIGHT so that the divisor is a whole number ▪ MOVE THE DECIMAL POINT IN THE DIVIDEND THE SAME NUMBER OF DECIMAL PLACES TO THE RIGHT ❖ Divide the decimals as though they were whole numbers ❖ The decimal point in the answer should BE STRAIGHT ABOVE THE DECIMAL POINT IN THE DIVIDEND	▹ $.02\overline{)\,.25}$ $\begin{array}{r} 12.5 \\ 2\overline{)\,25.\mathbf{0}} \\ 2 \\ \hline 05 \\ 4 \\ \hline 1\,0 \\ 1\,0 \\ \hline 0 \end{array}$
To Multiply by Powers of 10 (shortcut)	❖ Move the DECIMAL POINT TO THE RIGHT the same number of places as there are zeros in the power of 10 ▪ Move to the right because the number should get bigger (Add zeros if needed)	▹ $67.6 \bullet 100$ $= 67.60$ $= 6760.$
To Divide by Powers of 10 (shortcut)	❖ Move the DECIMAL POINT TO THE LEFT the same number of places as there are zeros in the power of 10 ▪ Move to the left because the number should get smaller (Add zeros if needed)	▹ $67.6 / 1000$ $= 067.6$ $= .0676$

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5. Conversions

5.1. Percent to Decimal to Fractions			
	To Percent*	**To Decimal**	**To Fraction**
From Percent* ➡ 123%		Drop the % sign & divide by 100. (move the decimal point 2 digits to the left) 1.23	Write the % value over 100. Always reduce. 123/100 ▪ If there is a decimal point, multiply numerator & denominator by a power of 10 to eliminate it. $\frac{90.5}{100}=\frac{90.5\cdot 10}{100\cdot 10}=\frac{905}{1000}=\frac{181}{200}$ ▪ If there is a fraction part, write the percent value as an improper fraction $5\frac{5}{6}\%=\frac{5\frac{5}{6}}{100}=\frac{35}{6}\cdot\frac{1}{100}=\frac{35}{600}$
From Decimal ➡ 1.234	Multiply by 100 (move the decimal point 2 digits to the right) & attach the % sign 123.4%		Write number part. Put decimal part over place value of right most digit. Always reduce. $1\frac{234}{1000}=1\frac{117}{500}$
From Fraction ➡ $\frac{1}{6}$	Method 1 - To express as a <u>mixed number</u> – multiply by 100 $\frac{1}{6}\bullet\frac{100}{1}=\frac{1\bullet\cancel{100}^{50}}{\cancel{6}_{3}}$ $=16\frac{2}{3}\%$ Method 2 - To express as a <u>decimal</u> – convert to decimal & multiply by 100 $.167\bullet 100=16.7\%$	Perform long division $\begin{array}{r} 0.1666 \\ 6\overline{)1.0000} \\ \underline{6} \\ 40 \\ \underline{36} \\ 40 \\ \underline{36} \\ 40 \\ \underline{36} \\ 4 \end{array}$ .167	

"%" means "per hundred"

6. Algebra

6.1. Definitions		
Constant	▪ A NUMBER	▷ 5
Variable	▪ A LETTER which represents a number	▷ x
Coefficient	▪ A NUMBER associated with variable(s)	▷ $3x$
Term	▪ A COMBINATION of coefficients and variable(s), or a constant	▷ $-3x$
Like Terms	▪ Each variable (including the exponent) of the terms is exactly the same, but they don't have to be in the same order	▷ $-3x$ & $2x$ ▷ $-3x^2$ & $2x^2$ ▷ $-3xy$ & $2xy$
Linear Expression	▪ One or more terms put together by a "+" or "–" ▪ The variable is to the first power	▷ $-3x + 3$
Linear Equation	▪ Has an equals sign ▪ The variable is to the first power	▷ $-3x+ 3 = 1$

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6.2. Operations of Algebra		
Addition/ Subtraction $+\ -$ (Combining Like Terms)	Only <u>COEFFICIENTS</u> of <u>LIKE TERMS</u> are combined ▪ <u>Underline terms</u> (or use $\boxed{\text{box}}$ & circle) as they are combined	▷ $\underline{4}x-\underline{3}x=x$ ▷ $-3xy-2xy\boxed{+3}=-5xy+3$ ▷ $2xyz+3xy$ Can't be combined ▷ $2y^2+y$ Can't be combined
Multiplication/ Division $\bullet\ \div$	<u>COEFFICIENTS AND VARIABLES</u> of <u>ALL TERMS</u> are combined	▷ $(4x)(-3x)=-12x^2$ ▷ $(-3xy)(-2xy)(3)=18x^2y^2$ ▷ $(2xyz)(3xy)=6x^2y^2z$ ▷ $(2y^2)(y)=2y^3$

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6.3. Solving Equations with 1 Variable	
① **ELIMINATE FRACTIONS** ▪ Multiply both sides of the equation by the LCD	Ex. Solve $\frac{2x+1}{3}-x=0$ $\left(\frac{3}{1}\right)\left(\frac{(2x+1)}{3}-x\right)=0\left(\frac{3}{1}\right)$ $2x+1-3x=0$
② **REMOVE ANY GROUPING SYMBOLS SUCH AS PARENTHESES** ▪ Use Distributive Property	$2x+1-3x=0$
③ **SIMPLIFY EACH SIDE** ▪ Combine like terms	$2x+1-3x=0$ $-x+1=0$
④ **GET VARIABLE TERM ON ONE SIDE & CONSTANT TERM ON OTHER SIDE** ▪ Use Addition Property of Equality (moves the WHOLE term)	$-x+1-1=0-1$ $-x=-1$
⑤ **ELIMINATE THE COEFFICIENT OF THE VARIABLE** ▪ Use Multiplication Property of Equality (eliminates PART of a term)	$(-1)(-x)=(-1)(-1)$ $x=1$
⑥ **CHECK ANSWER ✓** ▪ Substitute answer for the variable in the **original** equation	$\frac{2(1)+1}{3}-(1)=0$ $0=0\,\surd$

Note: Occasionally, when solving an equation, the variable "cancels out":

- If the resulting equation is true (e.g. 5 = 5), then all real numbers are solutions.
- If the resulting equation is false (e.g. 5 = 4), then there are no solutions.

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6.4. Solving for a Specified Variable

① **CIRCLE THE SPECIFIED VARIABLE**	Ex. Solve $T_F = \frac{5}{9}T_C + 32$ for T_C, where T_C is the temperature is Celsius and T_F is the temperature in Fahrenheit
② **TREAT THE SPECIFIED VARIABLE AS THE ONLY VARIABLE IN THE EQUATION & USE THE STEPS FOR SOLVING LINEAR EQUATIONS** 1. Eliminate fractions 2. Remove parenthesis 3. Simplify each side 4. Get variable term on one side & constant term on other side (use addition/subtraction) 5. Eliminate the coefficient of the variable (use multiplication/division) 6. Check answer ✓	$T_F - 32 = \frac{5}{9}T_C + 32 - 32$ $\left(\frac{9}{5}\right)(T_F - 32) = \left(\frac{9}{5}\right)\frac{5}{9}T_C$ $\left(\frac{9}{5}\right)(T_F - 32) = T_C$

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6.5. Expressions vs. Equations		
	Expressions	**Equations**
Definition	One or more terms put together by a "+" or "–" Ex. x^2+2x+1	Expression=Expression Ex. $x^2+2x+1=0$
Equivalent	Evaluate both for $x=2$ & get same answer Ex. x^2+2x+1 $=x^2+3x-x+1$	Same solution Ex. $x^2+2x+1=0$ $\times 2$ $2x^2+4x+2=0$
Expand Opposite of factoring – rewrite without parenthesis	Ex. $2(x+1)$ $=2x+2$	Often used in solving equations
Factor Opposite of expanding **–** rewrite as a product of smaller expressions	Ex. $2x+2$ $=2(x+1)$	Often used in solving equations
Cancel Only within a fraction in factored form	Ex. $\frac{\cancel{2}(x+2)}{\cancel{2}}+3$ $=x+2+3$	Often used in solving equations
Simplify/ Evaluate/ Add/Subtract/ Multiply/Divide An equalivent expression with a smaller number of parts	Cancelling common factors (top & bottom) & collecting like terms Ex. $\frac{\cancel{2}(x+2)}{\cancel{4}^{2}}+3$ $=\frac{x+2}{2}+\frac{6}{2}=\frac{x+8}{2}$ *~denominators stay when adding fractions*	Often used in solving equations *~denominators are eliminated when simplifying equations*
Evaluate for a number Substitute the given number (put it in parenthesis) & simplify	Ex. Evaluate x^2 for $x=-2$ –base is -2, not 2! $(-2)^2$ $=(-2)\bullet(-2)$ $=4$	Used to check answers Ex. Evaluate $0=x+2$ for $x=-2$ $0=(-2)+2$ $0=0$ √ so -2 is a solution
Solve		Find all possible values of the variable(s) Ex. $x+2=0$ $x=-2$

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7. Word Problems

7.1. Vocabulary		
Addition	▪ The *sum* of a and b ▪ The *total of* a, b, and c ▪ 8 *more than* a ▪ a *increased* by 3	$a + b$ $a + b + c$ $a + 8$ $a + 3$
Subtraction	▪ a *subtracted from* b ▪ The *difference of* a and b ▪ 8 *less than* a ▪ a *decreased by* 3	$b - a$ $a - b$ $a - 8$ $a - 3$
Multiplication	▪ 1/2 *of* a ▪ The *product of* a and b ▪ *twice* a ▪ a *times* 3	$(1/2)a$ $a \bullet b$ $2a$ $3a$
Division	▪ The *quotient of* a and b ▪ 8 *into* a ▪ a *divided by* 3	$a \div b$ $a/8$ $a/3$
General	▪ *Variable* words ▪ *Multiplication* words ▪ *Equals* words	*what, how much, a number* *of* *is, was, would be*
Percent Word Problems	▪ 50% *of* 60 *is what* ▪ 50% *of what is* 30 ▪ *What* % *of* 60 *is* 30	$50\% \bullet 60 = a$ $50\% \bullet a = 30$ $(a\%) \bullet 60 = 30$

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7.2. Formulas	
Commission	▪ Commission = commission rate • sales amount
Sales Tax **Total Price**	▪ *Sales tax* = sales tax rate • purchase price ▪ Total price = purchase price + *Sales tax*
Amount of Discount **Sale Price**	▪ *Amount of discount* = discount rate • original price ▪ Sale price = original price – *Amount of discount*
Simple Interest	▪ Simple interest = principal • interest rate • time
Percent increase/decrease	▪ $\text{Percent increase/decrease} = \left(\dfrac{\text{amount of increase/decrease}}{\textbf{original}\text{ amount}}\right) \bullet 100$

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7.3. Steps for Solving	
① **UNDERSTAND THE PROBLEM** ❖ As you use information, ~~cross~~ it out or underline it. ❖ Remember units!	Kevin's age is 3 years *more than twice* Jane's age. The *sum* of their ages is 39. How old are Kevin and Jane?
② **NAME WHAT x IS** ❖ Start your LET statement ❖ x can only be one thing ❖ When in doubt, choose the smaller thing	**Let** x = Jane's age (years)
③ **DEFINE EVERYTHING ELSE IN TERMS OF x**	$2x + 3$ = Kevin's age
④ **WRITE THE EQUATION**	Kevin's age + Jane's age = 39 $\underline{2x+3} + \underline{x} = 39$
⑤ **SOLVE THE EQUATION**	$3x + 3 = 39$ $(\cancel{-3}) + 3x + \cancel{3} = 39 + (-3)$ $\left(\frac{1}{3}\right)\frac{3x}{1} = \frac{36}{1}\left(\frac{1}{3}\right)$ $x = 12$
⑥ **ANSWER THE QUESTION** ❖ Answer must include units!	Jane's age = 12 years Kevin's age = 2(12) + 3 = 27 years
⑦ **CHECK**	$12 + 27 = 39$ √

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7.4. Ratios & Proportions		
Rate **Ratio**	▪ The quotient of two quantities ▪ Used to compare different kinds of rates	$\frac{1\text{ person}}{100\text{ people}}, \frac{2\text{ gallons}}{50\text{ miles}}$
Proportion	▪ A statement that TWO RATIOS or RATES are EQUAL ▪ On a map 50 miles is represented by 25 inches. 10 miles would be represented by how many inches?	$\frac{10\text{ (miles)}}{x\text{ (inches)}} = \frac{50\text{ (miles)}}{25\text{ (inches)}}$
Cross Product (shortcut)	▪ Method for solving for x in a proportion ▪ Multiply diagonally across a proportion ▪ If the cross products are equal, the proportion is true. If the cross products are not equal, the proportion is false	$\frac{10}{x} = \frac{50}{25}$ $10 \bullet 25 = 50 \bullet x$ $250 = 50x$ $\frac{250}{50} = x$ $5 = x$

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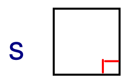
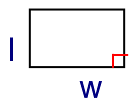
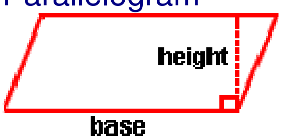
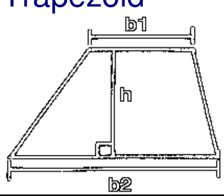
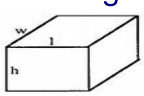
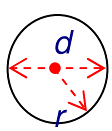
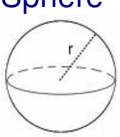
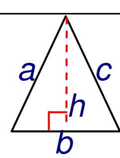
7.5. Geometry		
Terminology	Perimeter/ Circumference	▪ Measures the length around the outside of the figure (RIM); the answer is in the same *units* as the sides
	Area	▪ Measures the size of the enclosed region of the figure; the answer is in *square units*
	Surface Area	▪ Measures the outside area of a 3 dimensional figure; the answer is in *square units*
	Volume	▪ Measures the enclosed region of a 3 dimensional figure; the answer is in *cubic units*
Formulas	Square 	▪ PERIMETER: $P = 4s$ ▪ AREA: $A = s^2$ (s = side)
	Rectangle 	▪ PERIMETER: $P = 2l + 2w$ ▪ AREA: $A = lw$ (l = length, w = width)
	Parallelogram 	▪ DEFINITION: a four-sided figure with two pairs of parallel sides. ▪ PERIMETER: $P = 2h + 2b$ ▪ AREA: $A = hb$ (h = height, b = base)
	Trapezoid 	▪ DEFINITION: a four-sided figure with one pair of parallel sides ▪ PERIMETER: $P = b_1 + b_2 +$ other two sides ▪ AREA: $A = ((b_1 + b_2) / 2)h$ (h = height, b = base)
	Rectangular Solid 	▪ SURFACE AREA: $A = 2hw + 2lw + 2lh$ ▪ VOLUME: $V = lwh$ (l = length, w = width, h = height)
	Circle 	▪ CIRCUMFERENCE: $C = 2\pi r$ ▪ AREA: $A = \pi r^2$ $\left(r = \text{radius},\ d = \text{diameter},\ r = \frac{1}{2} d\right)$
	Sphere 	▪ SURFACE AREA: $A = \pi d^2$ ▪ VOLUME: $V = \frac{4}{3}\pi r^3$
	Triangle 	▪ PERIMETER: $P = a + b + c$ ▪ AREA: $A = \frac{1}{2}bh$

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