Part 3 - Intermediate Algebra Summary

Contents

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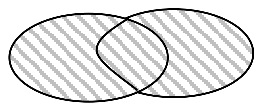
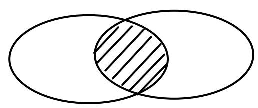
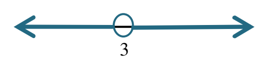
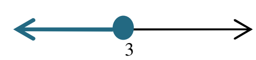
1 Definitions

1.1 Sets		
Set	▪ Any collection of things. It can be finite or infinite	▹ A set of my favorite fruits ▹ The set of integers between 1 and 5 ▹ A set of ordered pairs
Set Notation $\{\ \}$	▪ Expresses sets (usually finite sets)	▹ {apples, oranges, strawberries} ▹ $\{2,3,4\}$ ▹ $\{(1,2),(2,3),(3,4)\}$
Union $\cup$ Or	The union of 2 sets, A and B, is the set of elements that belong to either of the sets 	▹ $\{2,4,6\}\cup\{6,8,10\}=\{2,4,6,8,10\}$
Intersection $\cap$ And	▪ The intersection of 2 sets, A and B, is the set of all elements common to both set. 	▹ $\{2,4,6\}\cap\{6,8,10\}=\{6\}$
Null Set $\varnothing, \{\ \}$	▪ "Empty Set" ▪ Contains no members	▹ $\{2,4,6\}\cap\{8,10\}=\{\ \}$
Number Lines **Interval Notation** **Set Builder Notation**	▪ 3 unique methods of expressing sets (finite or infinite) ▪ All three methods are equally good, but read the directions carefully and answer in the correct format ▪ See *Number Lines & Interval Notation - MA091* ▪ Set builder notation looks like $\{x \mid x \neq 3\}$. It is read "The set of all x such that x is not equal to 3"	▹ $x<3 \cup x>3$ Number line:  Interval notation: $(-\infty,3)\cup(3,\infty)$ Set builder notation: $\{x \mid x \neq 3\}$ ▹ $x \leq 3$ Number line:  Interval notation: $(-\infty,3]$ Set builder notation: $\{x \mid x \leq 3\}$

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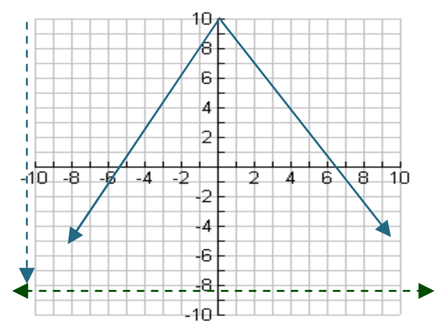
1.2 Relations, Domain & Range

Relation	▪ A set of ordered pairs. ▪ Equations in 2 variables are also relations since they define a set of ordered pair solutions.	▹ Relation: $\{(0,2),(1,2)\}$ Domain: $\{0,1\}$ Range: $\{2\}$
Domain (independent variables)	▪ The set of all possible *x*-coordinates for a given relation (inputs) ▪ Beware of values in the domain which create "impossibilities" – e.g. those that make a denominator equal 0, those that make a radicand negative ▪ To determine domain from a graph, project values onto the *x*-axis	▹ Relation: $g(x)=\dfrac{1}{x-2}$ $x-2=0 \rightarrow x=2$ Domain: $\{x \mid x \neq 2\}$ ▹ Relation: $g(x)=\sqrt{x-2}$ $x-2 \geq 0 \rightarrow x \geq 2$ Domain: $\{x \mid x \geq 2\}$ ▹ Relation - see graph below:
Range (dependent variables)	▪ The set of all possible *y*-coordinates for a given relation (outputs) ▪ To determine range from a graph, project values onto the *y*-axis	

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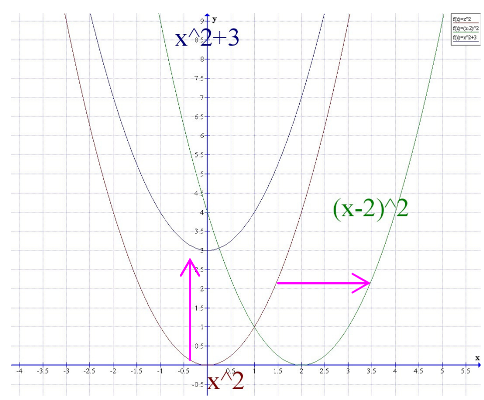
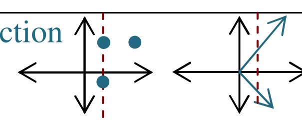
1.3 Functions

Function	▪ A set of ordered pairs that assign to each x-value exactly one y-value ▪ All functions are relations, but not all relations are functions. ▪ Linear equations are always functions	Relation Function 
Function Notation *f(x)*	▪ Read "function of x" or "f of x" ▪ $f(x)$ is another way of writing y	▷ $y = x+1$ may be written $f(x) = x+1$ ▷ (x,y) may be written $(x,f(x))$
	▪ Any linear equation that describes a function can be written in this form 1. Solve the equation for y 2. Replace y with $f(x)$	▷ Given : $x + y = 1$ 1. $y = -x + 1$ 2. $f(x) = -x + 1$
Evaluate *f*(*x*)	▪ Use whatever expression is found in the parentheses following the f to substitute into the rest of the equation for the variable x, then simplify completely. ▪ $f(x)$ can be expressed as an ordered pair $(x,f(x))$ ▪ For any function $f(x)$, the graph of $f(x) + k$ is the same as the graph of $f(x)$ shifted k units upward if k is positive and $\|k\|$ units downward if k is negative.	▷ $f(x) = x^2$ Evaluate the function $f(x)$ for $x = x-2$ $f(x-2) = (x-2)^2$ $= x^2 - 4x + 4$  *Note* • *f(x-2) shifts f(x) to the right by 2* • *Note f(x)+3 shifts f(x) to the up by 3*
Vertical Line Test	▪ If a vertical line can be drawn so that it intersects a graph more than once, the graph is not a function	Not a function 

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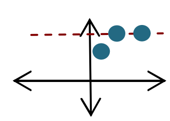
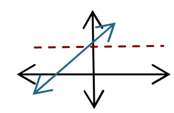
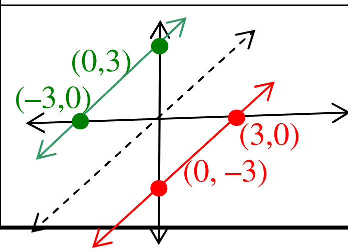
1.4 Inverse Functions

One-To-One Function	▪ In addition to being a function, every element of the range maps to a unique element in the domain	Not one-to-one $1 \rightarrow 4$ $2 \rightarrow 5$ $3 \nearrow$ One-to-one $1 \leftrightarrow 4$ $2 \leftrightarrow 5$ $3 \leftrightarrow 6$
Horizontal Line Test	▪ If a horizontal line can be drawn so that it intersects a graph more than once, the graph is not a one-to-one function	Not one-to-one  $\{(1,4),(2,5),(3,5)\}$ One-to-one  $f(x) = x + 3$
Inverse Function f^{-1}	▪ A way to get back from y to x ▪ The inverse function does the inverse operations of the function in reverse order ▪ f^{-1} denotes the inverse of the function f. It is read "f inverse" ▪ The symbol does not mean $\frac{1}{f}$	$f(x) = x + 3$ x | y: -3 | 0; 0 | 3; 1 | 4 $f^{-1}(x) = x - 3$
To Find the Inverse of a One-to-one Function *f*(*x*)	1. Replace $f(x)$ with y 2. Interchange x and y 3. Solve for the new y 4. Replace y with $f^{-1}(x)$ 5. Check using Compositions of Functions or Graphing	Find the inverse of $f(x) = x + 3$ 1. $y = x + 3$ 2. $x = y + 3$ 3. $x - 3 = y$ $y = x - 3$ 4. $f^{-1}(x) = x - 3$

Composition of Functions
$f \circ g$
$f(g(x))$
	▪ $f(g(x))$ is read "f of g" or "the composition of f and g". Evaluate the function g first, and then use this result to evaluate the function f
	▪ If functions are not inverses…
	  ▪ $f(g(x)) \neq g(f(x))$ -- order matters
	▪ If functions are inverses…
	  ▪ $f(f^{-1}(x)) = f^{-1}(f(x))$ -- order doesn't matter
	  ▪ $f^{-1}(f(x)) = x$ The function f^{-1} takes the output of $f(x)$, back to x
		Let $f(x) = x + 3$
		$f^{-1}(x) = x - 3$
		
		Find $f^{-1}(f(1))$
		$f(1) = (1) + 3 = 4$
		$f^{-1}(f(1)) = f^{-1}(4) = (4) - 3 = 1$
		
		Find $f(f^{-1}(1))$
		$f^{-1}(1) = (1) - 3 = -2$
		$f(f^{-1}(1)) = f(-2) = (-2) + 3 = 1$

Graphing	▪ The graph of a function f and its inverse f^{-1} are mirror images of each other across the line $y = x$ ▪ If f & f^{-1} intersect, it will be on the line $y = x$ ▪ For calculator, use a square window	

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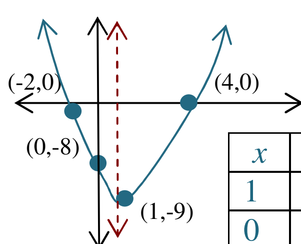
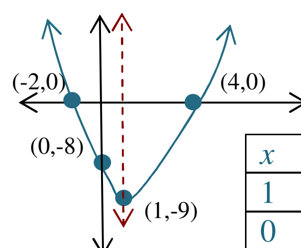
2 Quadratics

2.1 Standard vs. Easy to Graph Form			
		Standard Form $f(x) = ax^2 + bx + c$ Ex: $f(x) = x^2 - 2x - 8$	**Easy to Graph Form** $f(x) = a(x-h)^2 + k$ Ex: $f(x) = (x-1)^2 - 9$
Solution	▪ Parabola		
Vertex	▪ High or low point	$\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$ $= \left(\frac{2}{2(1)}, f(1)\right) = (1,-9)$	$(h, k) = (1, -9)$ *Note: h is the constant after the minus sign:* $f(x)=(x+1)^2 - 9$ becomes $f(x)=(x-(-1))^2-9$ & $h = -1$
Line of Symmetry	▪ Line which graph can be folder on so 2 halves match – vertical line thru vertex	$x = \frac{-b}{2a} = \frac{2}{2(1)} = 1$	$x = h$ $= 1$
Direction	▪ The parabola opens up if a > 0, down if a < 0	a is positive so parabola opens up	a is positive so parabola opens up
Shape	▪ If ξaξ>1, the parabola is steeper than $y = x^2$ ▪ If ξaξ< 1, the parabola is wider than $y = x^2$	$a = 1$, so parabola is the same shape as $y = x^2$	$a = 1$, so parabola is the same shape as $y = x^2$
***x*-intercept(s)** (roots/ zeros)	▪ Set $y = 0$ and solve for x ▪ If real roots exist, the line of symmetry parses exactly half-way between them	$0 = x^2 - 2x - 8$ $x = \frac{2 \pm \sqrt{(-2)^2 - 4(1)(-8)}}{2}$ $x = 4,\ -2$ $(4,0), (-2,0)$	$0 = (x-1)^2 - 9$ $\sqrt{9} = \sqrt{(x-1)^2}$ $\pm 3 = x - 1$ $x = 4, -2$ $(4,0), (-2,0)$
***y*-intercept**	▪ Set $x = 0$ and solve for y	$y = (0)^2 - 2(0) - 8$ $= -8$ $(0, -8)$	$y = ((0) - 1)^2 - 9$ $= -8$ $(0, -8)$
Graphing	▪ Use vertex & direction (in addition, can also include shape, roots & *y*-intercept) ▪ Plot points (plot vertex, 1 value to left of vertex & 1 value to right of vertex)	 x | y: 1 | –9; 0 | –8; 2 | –8	 x | y: 1 | –9; 0 | –8; 2 | –8
Converting Between Forms		To *Easy to Graph Form* 1. Complete the square $y + 8 + 1 = x^2 - 2x + 1$ $y + 9 = (x-1)^2$ 2. Solve for y	To *Standard Form* 1. Expand $y = x^2 - 2x + 1 - 9$ 2. Simplify

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2.2 Solving

Square Root Property If you can isolate the variable factor if $a^2 = b$, then $a = \pm\sqrt{b}$	1. Isolate the variable factor 2. Take the square root of both sides 3. Solve 4. Check	Ex $47{,}096 = 35{,}000(1+r)^2$ $\frac{47{,}096}{35{,}000} = \frac{35{,}000(1+r)^2}{35{,}000}$ $1.3456 = (1+r)^2$ $\pm\sqrt{1.3456} = (1+r)$ $-1 \pm \sqrt{1.3456} = r$
Factoring Only works when answers are integers	1. Set equation equal to 0 2. Factor 3. Set each factor containing a variable equal to 0 4. Solve the resulting equations & check	
"Completing the Square" & then using the "Square Root Property" Deriving the quadratic formula	1. If the coefficient of x^2 is not 1, divide both sides of the equation by the coefficient of x^2 (this makes the coefficient of x^2 equal 1) 2. Isolate all variable terms on one side of the equation 3. Complete the square for the resulting binomial. • Write the coefficient of the x term • Divide it by 2 (or multiply it by ½) • Square the result • Add result to both sides of the equation 4. Factor the resulting perfect square trinomial into a binomial squared 5. Use the *square root property* 6. Solve for x 7. Check	Ex: $2x^2 - 4x - 3 = 0$ 1. $x^2 - \frac{4}{2}x - \frac{3}{2} = 0$ 2. $x^2 - 2x = \frac{3}{2}$ 3. The coefficient of $x = -2$ $1/2 \cdot (-2) = -1$ $(-1)^2 = 1$ $x^2 - 2x + 1 = \frac{3}{2} + 1$ 4. $(x-1)^2 = \frac{5}{2}$ 5. $(x-1) = \pm\sqrt{\frac{5}{2}}$ 6. $x = 1 \pm \frac{\sqrt{5}}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = 1 \pm \frac{\sqrt{10}}{2}$
Quadratic Formula Works all the time (when answers are integer, real, or imaginary numbers)	1. Set equation equal to 0 2. Plug values into the quadratic formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ 3. Solve & check	
	▪ The discriminant tells the number and type of solutions. The discriminant is the radicand in the quadratic formula.	(see table below)

$b^2 - 4ac$	**Number & Type of Solutions**
Positive	2 real solutions
Zero	1 real solution
Negative	2 complex but not real solutions

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3 Higher Degree Polynomial Equations

3.1 Solving		
Where the exponent can be isolated $ax^3 = c$	1. Write the equation so that the variable to be solved for is by itself on one side of the equation 2. Raise each side *(not each term)* of the equation to a power so that the final power on the variable will be one. *(If both sides of an equation are raised to the same rational exponent, it is possible you will not get all solutions)* 3. Check answer	Ex $y^3 = 27$ $\left(y^{\frac{3}{1}}\right)^{\frac{1}{3}} = (27)^{\frac{1}{3}}$ $y = 3$ Ex $y^3 = 27$ - try factoring instead $y^3 - 27 = 0$ $(y-3)(y^2+3y+9) = 0$ $y = \frac{-3 \pm \sqrt{9-4(1)(9)}}{2}$ $y = \frac{-3 \pm 3i\sqrt{3}}{2}$ $y - 3 = 0$ $y = 3$
For equations that contain repeated variable expressions $ax^4 + bx^2 + c = 0$	▪ Apply the same steps as *Solving by Factoring & Zero Factor Property – MA091*	Ex $p^4 - 4p^2 + 4 = 0$ $(p^2-2)(p^2-2) = 0$ $(p^2-2) = 0$ $p^2 = 2$ $p = \pm\sqrt{2}$ $Check: (\sqrt{2})^4 - 4(\sqrt{2})^2 + 4 = 0$ $2^{4/2} - 4 \cdot 2^{2/2} + 4 = 0$ $4 - 8 + 4 = 0 \surd$ $Check: (-\sqrt{2})^4 - 4(-\sqrt{2})^2 + 4 = 0$ $2^{4/2} - 4 \cdot 2^{2/2} + 4 = 0$ $4 - 8 + 4 = 0 \surd$

4 Polynomial Division

Long Division	▪ To divide one polynomial by another ▪ Polynomial division is similar to integer division. However, instead of digit by digit, polynomial division proceeds term by term. ▪ In polynomial division, the remainder must be 0 -or- of a smaller degree than the divisor.	Ex $\dfrac{2x^3-x^2-8x-1}{x-2}$ $\begin{array}{r\|rrrr} & 2x^2 & 3x & -2 & \\ \hline x-2 & 2x^3 & -x^2 & -8x & -1 \\ & 2x^3 & -4x^2 & \downarrow & \downarrow \\ \hline & & 3x^2 & -8x & \\ & & 3x^2 & -6x & \\ \hline & & & -2x & -1 \\ & & & -2x & +4 \\ \hline & & & & -5 \end{array}$
Long Division Steps	1. Write both polynomials in order of descending degree. Insert $0x^n$ for all missing terms (even the constant). 2. Divide the leading term of the dividend by the leading term of the divisor to get the first term of the quotient (the coefficient may not be an integer). 3. Multiply the quotient term by the divisor & ***subtract*** the product from the dividend; the difference should have smaller degree than the original dividend. 4. Repeat, using the difference as the new dividend, until the next "new dividend" is 0 (the divisor is a factor of the dividend) or the new dividend has degree strictly smaller than the degree of the divisor (this last new dividend is the remainder).	Quotient $2x^2+3x-2+\dfrac{-5}{x-2}$
Synthetic Division	▪ A faster, slightly trickier way of dividing a polynomial by a binomial of the form $x-a$	Ex $\dfrac{2x^3-x^2-8x-1}{x-2}$ $\begin{array}{r\|rrrr} 2 & 2 & -1 & -8 & -1 \\ & \downarrow & 4 & 6 & -4 \\ \hline & 2 & 3 & -2 & \boxed{-5} \end{array}$
Synthetic Division Steps	1. In line 1, write the potential root (a if dividing by $x-a$). To the right on the same line, write the coefficients of the polynomials in descending degree. Insert 0 for all missing terms (even the constant). 2. Leaving space for line 2, draw a horizontal line under the coefficients. Copy the leading coefficient into line 3 under the horizontal line 3. Multiply that entry in line 3 by a and write the result in line 2, under the second coefficient. The first position of line 2 is blank 4. ***Add*** the numbers in the second position of lines 1 & 2, write in line 3 5. Repeat – multiply the new entry of line 3 by a, write in next position in line 2, add entries in lines 1 & 2, write in line 3- until done. 6. The last entry in line 3 is the remainder. The rest of line 3 represents the coefficients of the quotient, in descending order of degree. The degree of the quotient is one less than the degree of the dividend.	Quotient $2x^2+3x-2+\dfrac{-5}{x-2}$
Remainder Theorem	▪ If $f(x)$ is a polynomial, then the remainder from dividing $f(x)$ by $x-a$ is the value $f(a)$ ▪ You can also get the value of $f(a)$ with substitution	$f(2)=$ $f(2)=2(2)^3-(2)^2-8(2)\text{-}1$ $=16-4-16-1=-5$

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5 Complex Fractions		
Definition	▪ A rational expression whose numerator, denominator, or both contain one or more rational expressions	Ex. $\dfrac{\frac{x+2}{x}}{x-\frac{4}{x}}$
Simplifying: Method 1	1. Multiply the numerator and the denominator of the complex fraction by the LCD of the fractions in both the numerator and the denominator. 2. Simplify	Ex. $\dfrac{\left(\frac{x+2}{x}\right)\cdot x}{\left(x-\frac{4}{x}\right)\cdot x}=\dfrac{x+2}{x\cdot x-\frac{4}{x}\cdot x}$ $=\dfrac{x+2}{x^2-4}$ $=\dfrac{x+2}{(x-2)(x+2)}$ $=\dfrac{1}{x-2}$
Simplifying: Method 2	1. Simplify the numerator and the denominator of the complex fraction so that each is a single fraction 2. Perform the indicated division by multiplying the numerator of the complex fraction by the reciprocal of the denominator of the complex fraction 3. Simplify if possible	Ex. $\dfrac{\frac{x+2}{x}}{x-\frac{4}{x}}=\dfrac{\frac{x+2}{x}}{\frac{x^2-4}{x}}$ $=\dfrac{x+2}{x}\cdot\dfrac{x}{(x+2)(x-2)}$ $=\dfrac{1}{x-2}$

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6 Radicals

6.1 Expressions		
Like Radicals	▪ Same index and same radicand	▹ $\sqrt{6}$ & $5\sqrt{6}$
Add/Subtract *Like radicals only!*	▪ Same as polynomial expressions. Treat radicals as variables. ▪ Unlike radicals can only be combined under multiplication & division	▹ $5\sqrt{6}+2\sqrt{6}=(5+2)\sqrt{6}=7\sqrt{6}$ ▹ $5\sqrt{2}+2\sqrt{3}$ Can't be combined
Multiply/Divide *Like Indexes*	▪ Same as polynomial expressions. Treat radicals as variables.	▹ $(4\sqrt{x}-3)(5\sqrt{x}+2)$ $=20x+8\sqrt{x}-15\sqrt{x}-6$ $=20x-7\sqrt{x}-6$
Multiply/Divide *Unlike Indexes & Unlike Radicands*	1. Change to rational form 2. Write as equivalent expressions with like denominators – use the LCM of the original indices 3. Combine using the product rule	▹ $\sqrt{2}\cdot\sqrt[3]{3}=2^{\frac{1}{2}}\cdot 3^{\frac{1}{3}}$ $=2^{\frac{3}{6}}\cdot 3^{\frac{2}{6}}$ $=\sqrt[6]{8\cdot 9}$ $=\sqrt[6]{72}$
Conjugate	▪ To rationalize a numerator or denominator that is a sum or difference of two terms, use the conjugate. The conjugate of a + b is a - b	▹ $\sqrt{6}+2$ and $\sqrt{6}-2$ are conjugutes
Rationalize the Denominator/ Numerator	▪ Rewrite a radical expression without a radical in the denominator or without a radical in the numerator. ▪ If the radical expression to be rationalized is a monomial… 1. Write the radicand in power form 2. Multiply by "a clever form a 1" so that the power of the radicand will equal the index. 3. Simplify ▪ If the radical expression to be rationalized is a binomial… 1. Multiply expression by the conjugate 2. Simplify ▪ *Note: For MathXL you must rationalize the denominator of **all** answers unless otherwise specified*	Rationalize monomial denominator: Ex: $\sqrt[4]{\frac{1}{4}}$ 1. $=\frac{1}{\sqrt[4]{2^2}}$ 2. $=\frac{1}{\sqrt[4]{2^2}}\cdot\frac{\sqrt[4]{2^2}}{\sqrt[4]{2^2}}$ $=\frac{\sqrt[4]{4}}{\sqrt[4]{2^4}}$ 3. $=\frac{\sqrt[4]{4}}{2}$ Rationalize binomial denominator: Ex: $\frac{2}{3\sqrt{2}+4}=\frac{2}{3\sqrt{2}+4}\cdot\frac{3\sqrt{2}-4}{3\sqrt{2}-4}$ $=\frac{2(3\sqrt{2}-4)}{18-16}$ $=3\sqrt{2}-4$

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6.2 Solving

Where the radical can be isolated $\sqrt{x+a}=b$	1. Convert radicals to equivalent exponential form 2. Write the equation so that one radical is by itself on one side of the equation 3. Raise each side *(not each term)* of the equation to a power equal to the index of the radical. (*This will get rid of the radical which you isolated in step 2.)* 4. Simplify *(Sometimes when you are squaring a side of an equation, you end up with a binomial squared, you must remember to use FOIL).* 5. If the equation still contains a radical, repeat Steps 1 and 2. If not, solve. 6. Check all proposed solutions in the original equation *(If both sides of an equation are raised to the same power, you often get extra solutions)*	▷ $x-\sqrt{x}-6=0$ $x-x^{\frac{1}{2}}-6=0$ $x-6=x^{\frac{1}{2}}$ $(x-6)^2=\left(x^{1/2}\right)^2$ $x^2-12x+36=x$ $x^2-13x+36=0$ $(x-9)(x-4)=0$ $x=9,4$ $Check:(9)-\sqrt{(9)}-6=0\surd$ $Check:(4)-\sqrt{(4)}-6=0\times$ ▷ $\sqrt[3]{4x+7}+5=3$ $(4x+7)^{1/3}=-2$ $\left((4x+7)^{1/3}\right)^3=(-2)^3$ $4x+7=-8$ $x=\frac{-15}{4}$ $Check:\sqrt[3]{4\left(\frac{-15}{4}\right)+7}+5=3$ $\sqrt[3]{-8}+5=3\surd$
Where the equation contains repeated variable expressions $x^{2/3}+x^{1/3}+c=0$	▪ Apply the same steps as *Solving by Factoring & Zero Factor Property – MA091*	▷ $x^{2/3}-5x^{1/3}+6=0$ $(x^{1/3}-2)(x^{1/3}-3)=0$ $(x^{1/3}-2)=0$ $x^{1/3}=2$ $\left(x^{1/3}\right)^3=2^3$ $x=8$ $(x^{1/3}-3)=0$ $x^{1/3}=3$ $\left(x^{1/3}\right)^3=3^3$ $x=27$

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7 Complex Numbers (a + b*i*)

Imaginary Unit *i*	• The number whose square is -1	$i^2 = -1$ and $i = \sqrt{-1}$
To Write with *i* Notations	1. Write each number in terms of the imaginary unit *i* 2. Simplify	▷ $\sqrt{-16} = \sqrt{-1}\sqrt{16} = 4i$ ▷ $\sqrt{-5}\cdot\sqrt{-4} = i\sqrt{5}\cdot i\sqrt{4}$ $= i^2\sqrt{20} = -2\sqrt{5}$ *Note: The product rule for radicals does not hold true for imaginary numbers* $\sqrt{-5}\cdot\sqrt{-4} \neq \sqrt{20}$
Complex Numbers	• All numbers *a* + *bi* where *a* and *b* are real • Complex numbers are all sums and products of real and imaginary numbers	▷ $5+4i$
To Add or Subtract	• Add or subtract their real parts and then add or subtract their imaginary parts	▷ $(-3+2i)-(7-4i)$ $= -10+6i$
To Multiply	• Multiply as though they are binomials	▷ $(-3+2i)(7-4i)$ $= -21+26i+14i+8$ $= -13+40i$
Complex Conjugate	• *a* + *bi* and *a* - *bi*	▷ $5+4i$ and $5-4i$
To Divide	1. Multiply the numerator and the denominator by the conjugate of the denominator 2. The result is written with two separate parts – real & imaginary	▷ $\dfrac{4}{2-i} = \dfrac{4}{2-i}\cdot\dfrac{2+i}{2+i}$ $= \dfrac{8+4i}{4+1}$ $= \dfrac{8}{5}+\dfrac{4}{5}i$
To Compute Powers	1. Use power rules to break difficult to compute exponents into simpler, easy to compute exponents 2. If necessary, rationalize the denominator	$i^1 = i$ $i^5 = i$ $i^9 = i$ $i^2 = -1$ $i^6 = -1$ $i^{10} = -1$ $i^3 = -i$ $i^7 = -i$ $i^{11} = -i$ $i^4 = 1$ $i^8 = 1$ $i^{12} = 1$ ▷ $i^{22} = i^{20}\cdot i^2 = \left(i^4\right)^5\cdot i^2$ $= 1^5\cdot(-1) = 1\cdot(-1) = -1$ ▷ $i^{-9} = \dfrac{1}{i^9} = \dfrac{1}{i}\cdot\dfrac{i}{i} = \dfrac{i}{-1} = -i$

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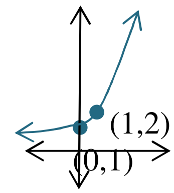
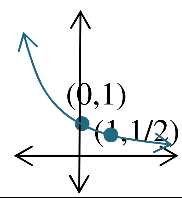
Part 3 - Intermediate Algebra Summary

8 Exponential

<table>
<tr><th colspan="3">8.1 Basics</th></tr>
<tr><td>Exponential Functions</td><td>▪ Where the variable is the exponent</td><td>▹ $f(x) = 2^x$</td></tr>
<tr><td>Characteristics of Exponential Functions</td><td>▪ One-to-one (inverses of exponential functions are logarithmic functions)
▪ y-intercept always (0,1)
▪ No x-intercept, instead has a x-axis asymptote (keeps getting closer to the x-axis, but never gets there)
▪ Graph always contains $(1,b)$
▪ Domain: $(-\infty,\infty)$ (x is a real number)
▪ Range: $(0,\infty)$ ($y > 0$)
▪ $b > 0, \quad b \neq 1$</td><td>Exponential Growth
$f(x) = b^x$, for $b > 1$
Ex: $f(x) = 2^x$
<table><tr><th>x</th><th>y</th></tr><tr><td>–2</td><td>1/4</td></tr><tr><td>0</td><td>1</td></tr><tr><td>1</td><td>2</td></tr><tr><td>2</td><td>4</td></tr></table>
Exponential Decay
$f(x) = b^x$, for $0 < b < 1$
Ex: $f(x) = \left(\frac{1}{2}\right)^x$
<table><tr><th>x</th><th>y</th></tr><tr><td>–2</td><td>4</td></tr><tr><td>0</td><td>1</td></tr><tr><td>1</td><td>1/2</td></tr><tr><td>2</td><td>1/4</td></tr></table></td></tr>
<tr><td>Uniqueness of b^x</td><td>▪ If $b > 0$ and $b \neq 1$, then
$b^x = b^y$ is equivalent to
$x = y$</td><td>$3^x = 3^4$
$x = 4$</td></tr>
<tr><td>Shifted Exponential Function</td><td>▪ $f(x) = 2^{x-3}$ (moves function to right 3)
▪ $f(x) = 2^x - 4$ (moves function down 4)</td><td></td></tr>
</table>

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8.2 Solving		
When you can write the bases the same $a^x = \left(a^2\right)^3$	▪ If you can write the bases the same: 1. Get common bases 2. Set exponents equal (uniqueness of b^x) 3. Solve for x 4. Check in original equation ▪ If you can't write the bases the same, then solve using logarithms.	Ex $27^{4x} = 9^{x+3}$ $(3^3)^{4x} = (3^2)^{x+3}$ $3^{12x} = 3^{2x+6}$ $12x = 2x + 6$ $10x = 6$ $x = \frac{6}{10} = \frac{3}{5}$ Check $27^{\frac{4}{1}\left(\frac{3}{5}\right)} = 9^{\left(\frac{3}{5}\right)+\frac{3}{1}}$ $27^{\left(\frac{12}{5}\right)} = 9^{\frac{3}{5}+\frac{15}{5}}$ $(3^3)^{\left(\frac{12}{5}\right)} = (3^2)^{\frac{18}{5}}$ $3^{\frac{36}{5}} = 3^{\frac{36}{5}}\ \surd$
Where You Can't Write the Bases the Same $a^x = b^3$	**Method 1 (Convert to log form)** 1. Get base and exponent alone on one side of equals sign 2. Convert to log form 3. Change to base 10 4. Compute 5. Check	Ex $2^x = 7$ $\log_2 7 = x$ (definition of logarithms) $\frac{\log 7}{\log 2} = x$ (exact answer) $2.81 \approx x$ (approximate answer) Check $2^{\left(\frac{\log 7}{\log 2}\right)} = 7\ \surd$
	Method 2 ("Common log" both sides) 1. Get base and exponent alone on one side of equals sign 2. "log" both sides 3. Power rule 4. Solve for variable 5. Compute 6. Check	Ex $2^x = 7$ $\log 2^x = \log 7$ (log both sides) $x \log 2 = \log 7$ (power rule) $x = \frac{\log 7}{\log 2}$ (exact answer) $x \approx 2.81$ (approximate answer) Check $2^{\left(\frac{\log 7}{\log 2}\right)} = 7\surd$
	Method 3 ("log of a base" both sides) 1. Get base and exponent alone on one side of equals sign 2. "$\log_b$" both sides 3. Log of a base rule 4. Compute 5. Check	Ex $2^x = 7$ $\log_2 2^x = \log_2 7$ ($\log_b$ both sides) $x = \log_2 7$ (def. of logarithms) $x = \frac{\log 7}{\log 2}$ (exact answer) $x \approx 2.81$ (approximate answer) Check $2^{\left(\frac{\log 7}{\log 2}\right)} = 7\surd$

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9 Logarithms

9.1 Basics

Definition of Logarithms

- Logarithms are exponents

$$B^E = N$$
$$\log_B N = E$$

"A base raised to an exponent equals a number $\Leftrightarrow$ The log to the base B of a number equals an exponent"

Ex $\mathbf{log}_2 8 = 3$

$2^3 = 8$

Ex $2^{-2} = \frac{1}{4}$

$\mathbf{log}_2 \frac{1}{4} = -2$

How to graph Logarithmic Functions

1. Replace $f(x)$ with y
2. Write the equivalent exponential equation
3. Make an x-y chart (find ordered pair solutions)
4. Plot the points
5. Connect the points with a smooth curve

Characteristics of logarithmic functions

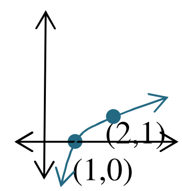
- One-to-one (inverses of logarithmic functions are exponential functions)
- x-intercept always (1,0)
- No y-intercept, instead has a y-axis asymptote (keeps getting closer to the y-axis, but never gets there)
- Graph always contains $(b, 1)$
- Domain: $(0, \infty)$ $(x > 0)$
- Range: $(-\infty, \infty)$ (y is a real number)
- $b > 0, \quad b \neq 1$

$f(x) = \log_b x$, for $b > 1$

Ex: $f(x) = \log_2 x$

$2^y = x$

x	y
1/4	−2
1	0
2	1
4	2

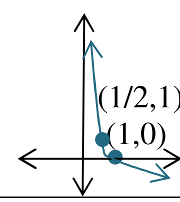


$f(x) = \log_b x$, for $0 < b < 1$

Ex: $f(x) = \log_{\frac{1}{2}} x$

$$\left(\frac{1}{2}\right)^y = x$$

x	y
4	−2
1	0
1/2	1
1/4	2



Common log
$\log x$

- If no base is indicated, the understood base is always 10

$$\log x = \log_{10} x$$

Ex $\mathbf{log}\, 100 =$

$\mathbf{log}_{10} 100 = 2$ (exact answer)

$10^2 = 100\surd$

e

- One of the most important constants in mathematics
- $e \approx 2.7182$

Ex: Continuously compounded interest

$A = Pe^{rt}$

Ex: The temperature of a cup of coffee

$f(t) = 70 + 137e^{-.06t}$

Natural log
$\ln x$

- A logarithm with base e

$$\ln x = \ln_e x$$

Ex $\mathbf{ln}\, 100 =$

$\mathbf{ln}_e 100 \approx 4.6052$ (approximate)

$e^{4.6052} \approx 100\surd$

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9.2 Properties

	Rule	Comment
Property of Equality (Allows you to "log" or "unlog" both sides of an equations. Useful in solving logarithmic and exponential equations)	$x = y$ is equivalent to $\log_b x = \log_b y$	Ex $3^x = 9$ $\log(3^x) = \log(9)$
Power Property	$\log_b \left(x^n\right) = n \log_b x$	Ex $\log(3^x) = \log(9)$ $x\log(3) = \log(9)$ $x = \dfrac{\log(9)}{\log(3)}$ $= 2$
Change of base (Allows you to use your calculator log10 button no matter what the base is.)	$\log_B A = \dfrac{\log A}{\log B}$	Ex $\log_2 5 = \dfrac{\log 5}{\log 2}$ $2^{\frac{\log 5}{\log 2}} = 5$ (def of logarithms) $\log 2^{\frac{\log 5}{\log 2}} = \log 5$ (log both sides) $\dfrac{\log 5}{\log 2}\log 2 = \log 5 \surd$
Log of 1	$\log_b 1 = 0$	Ex $\log_3 1 = 0$ $3^0 = 1 \surd$ (def of logs)
Log of the base	$\log_b b^x = x$	Ex $\log_{10} 10^2 = 2$ $10^2 = 10^2 \surd$ (def of logs)
Log as an exponent	$b^{\log_b x} = x$	Ex $10^{\log_{10} 2} = 2$ $\log_{10} 2 = \log_{10} 2 \surd$ (def of logs)
Product Property	$\log_b xy = \log_b x + \log_b y$ ~ multiplication on the "inside", addition on the "outside" ~ $\log_b(x+y)$ can't be simplified	Ex $\log_{10}(10\cdot 10) = \log_{10} 10 + \log_{10} 10$ $2 = 1 + 1$ (compute logs) $2 = 2 \surd$
Quotient Property	$\log_b \dfrac{x}{y} = \log_b x - \log_b y$ ~ division on the "inside", subtraction on the "outside" ~ $\log_b(x-y)$ can't be simplified	Ex $\log_{10}\left(\dfrac{1000}{10}\right) = \log_{10} 1000 - \log_{10} 10$ $2 = 3 - 1$ (compute logs) $2 = 2 \surd$

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9.3 Expressions		
Expansions	▪ To change something complicated into simple additions & subtractions.	Ex $\log \frac{a^3 b}{c^2} = \log a^3 b - \log c^2$ $= \log a^3 + \log b - \log c^2$ $= 3\log a + \log b - 2\log c$
Contractions	▪ To have one big log, useful in solving equations	Ex $2\log 3 + 3\log 2 = \log 3^2 + \log 2^3$ $= \log(9 \cdot 8)$ $= \log 72$
Evaluate	1. Use change of base rule – or – convert to exponential notation 2. Simplify if possible. 3. If a log can be computed exactly, compute it. Else it is more exact leave it “alone”. Sometimes you will be asked for an approximate solution; in that case, round the log values	Ex $\ln \sqrt[3]{e}$ $= \frac{\ln_e \sqrt[3]{e}}{\ln_e e}$ (change of base) $= \frac{\frac{1}{3}\ln_e e}{\ln_e e} = \frac{1}{3}$ Ex $\ln \sqrt[3]{e}$ $e^x = e^{\frac{1}{3}}$ (def of logs) $x = \frac{1}{3}$
Evaluate (given log values)	1. Expand expression into simple additions and multiplications 2. Plug in given log values	Ex If $\log_b 3 = .56$ and $\log_b 2 = .36$, evaluate $\log_b \sqrt{\frac{2}{3}}$ $\log_b \sqrt{\frac{2}{3}} = \log_b \frac{2^{\frac{1}{2}}}{3^{\frac{1}{2}}}$ $= \log_b 2^{\frac{1}{2}} - \log_b 3^{\frac{1}{2}}$ (quotient rule) $= \frac{1}{2}\log_b 2 - \frac{1}{2}\log_b 3$ (power rule) $= \frac{1}{2}(.56) - \frac{1}{2}(.36)$ $= -.1$

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9.4 Solving		
"*the answer*" is the variable $\log_a x = b$	1. Put all logarithm expressions on one side of the equals sign	Ex $\log_2 x = -\log_2(x-1)+1$ $\log_2 x + \log_2(x-1) = 1$
	2. Use the properties to simplify the equation to one logarithm statement on one side of the equals sign	$\log_2 x(x-1) = 1$ $\log_2(x^2 - x) = 1$
	3. Convert the equation to the equivalent exponential form	$2^1 = x^2 - x$
	4. Solve	$0 = x^2 - x - 2$ $0 = (x-2)(x+1)$ $x = 2, -1$
	5. Check	can't take the log of a negative number $x = 2$ Check $\log_2(2) = -\log_2((2)-1)+1$ $1 = 1\surd$
"*The base*" is the variable $\log_x a = b$	1. Convert to exponential form	Ex $\log_b 256 = \frac{4}{3}$ $b^{\frac{4}{3}} = 256$
	2. Solve - get the variable by itself by raising each side to a common exponent	$\left(b^{\frac{4}{3}}\right)^{\frac{3}{4}} = 256^{\frac{3}{4}}$ $b = 64$
	3. Check	$\frac{\log 256}{\log 64} = \frac{4}{3}\surd$
"*The exponent*" is the variable $\log_a b = x$	1. Compute using change of base formula	Ex $\log_2 3 = x$ $x = \frac{\log 3}{\log 2}$
	2. Check by converting to exponential form	$2^{\frac{\log 3}{\log 2}} = 3\surd$

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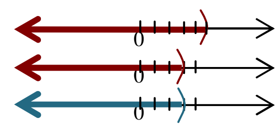
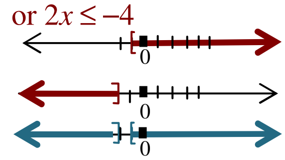
10 Inequalities

10.1 Compound Inequalities in 1 Variable		
Definition	▪ 2 inequalities joined by the words **and** or **or** ▪ 2 inequalities in one statement (compact form). Just a shorthand for 2 inequalities joined by the word **and**	▹ $-2 \le 3 - x$ and $3 - x \le 0$ ▹ $-2 \le 3 - x \le 0$ same as above
Solving	1. IF 2 INEQUALITIES JOINED BY THE WORD **AND**, solve each separately and take the **intersection** $\cap$ of the solution sets	Ex 1. Solve $x < 5$ and $x < 3$  $(-\infty, 3)$
	2. IF 2 INEQUALITIES JOINED BY THE WORD **OR**, solve each separately and take the **union** $\cup$ of the solution sets	Ex 2. Solve $x - 2 \ge -3$ or $2x \le -4$  $(-\infty, -2] \cup [-1, \infty)$
	3. IF 2 INEQUALITY SIGNS, solve for x in the middle (Can also be rewritten as 2 inequalities joined by the word **and**) ▪ Note: some compound inequalities have no solution; some have all real numbers as solutions	Ex 3. Solve $-2 \le 3 - x \le 0$ $-2 - 3 \le 3 - x - 3 \le 0 - 3$ $-5 \le -x \le -3$ $\frac{-5}{-1} \le \frac{-x}{-1} \le \frac{-3}{-1}$ $-5 \ge x \ge -3$ $\varnothing$

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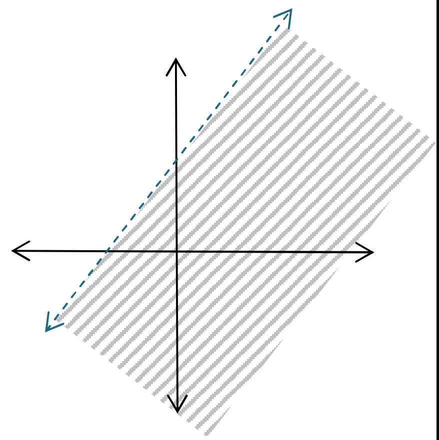
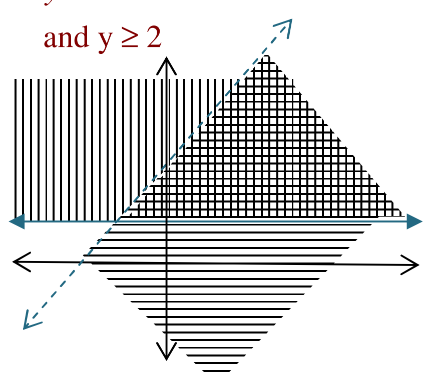
10.2 Linear Inequalities in 2 Variables

Definition	▪ Can be written in one of the forms: $Ax + By < C$ $\quad Ax + By \le C$ $Ax + By > C$ $\quad Ax + By \ge C$	▹ $y < x + 5$ 
Half-plane	▪ Every line divides a plane into 2 half-planes	
Boundary	▪ The line that divides the plane into two half-planes	
Solving	1. Graph the boundary line by graphing the related equation. ▪ Draw the line solid if the inequality symbol is $\le$ or $\ge$ ▪ Draw the line dashed if the inequality symbol is < or > 2. Choose a test point not on the line. Substitute its coordinates into the **original inequality**. Often (0, 0) makes an easy test point. 3. If the resulting inequality is true, shade the half-plane that contains the test point. If the inequality is not true, shade the half-plane that does not contain the test point.	

10.3 Systems of Linear Inequalities in 2 Variables

Definition	▪ 2 or more linear inequalities	▹ $y < x + 5$ and $y \ge 2$ 
Solving	1. Graph each inequality in the system. 2. The overlapping region is the solution of the system. ▪ Note: some systems have no solution; some have all real numbers as solutions	

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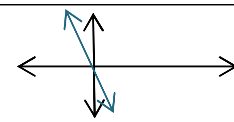
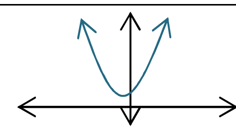
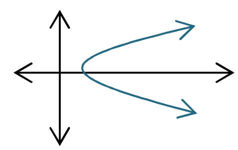
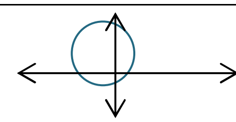
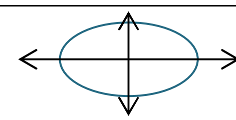
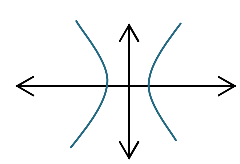
11 Systems of Non-Linear Equations

11.1 Conic Sections		
Conic Section	▪ The shape created by the intersection of a 3-dimensional cone and a plane cutting through it. ▪ A general conic section is the set of all solutions to the relation: $Ax^2 + By^2 + Cx + Dy + E = 0$	
If A = B = 0, then graph is a line	$2x + y = 0$ $y = -2x$ General Equation: $y = mx + b$	
If B = 0, then graph is a parabola	$0 = 3x^2 + 0y^2 + 2x - y + 4$ $y = 3x^2 + 2x + 4$ General Equation: $y = a(x-h)^2 + k$	
If A = 0, then graph is a parabola	$0 = 0x^2 + y^2 - x - 2y + 4$ $y^2 - 2y = x - 4$ $y^2 - 2y + 1 = x - 4 + 1$ $(y-1)^2 = x - 3$ $x = (y-1)^2 + 3$ General Equation: $x = a(y-k)^2 + h$	 $Y_1 = \sqrt{x-3} + 1$ $Y_2 = -\sqrt{x-3} + 1$
If A = B, then graph is a circle	$0 = x^2 + y^2 + 4x - 8y - 16$ $16 = x^2 + 4x + y^2 - 8y$ $16 + 4 + 16 = x^2 + 4x + 4 + y^2 - 8y + 16$ $36 = (x+2)^2 + (y-4)^2$ General Equation: $(x-h)^2 + (y-k)^2 = r^2$	 $Y_1 = 4 + \sqrt{-(x+2)^2 + 36}$ $Y_2 = 4 + -\sqrt{-(x+2)^2 + 36}$
If A ≠ B & they have the same sign, then graph is an ellipse	$0 = 4x^2 + 9y^2 + 0x + 0y - 36$ $36 = 4x^2 + 9y^2$ $1 = \frac{x^2}{9} + \frac{y^2}{4}$ General Equation: $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$	 $Y_1 = \frac{\sqrt{-4x^2 + 36}}{3}$ $Y_2 = \frac{-\sqrt{-4x^2 + 36}}{3}$
If A ≠ B & their signs are different, then graph is a hyperbola	$0 = 4x^2 - 9y^2 + 0x + 0y - 36$ $36 = 4x^2 - 9y^2$ $1 = \frac{x^2}{9} - \frac{y^2}{4}$ General Equation: $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$	 $Y_1 = 2\sqrt{\frac{x^2}{9} - 1}$ $Y_2 = -2\sqrt{\frac{x^2}{9} - 1}$

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<table>
<tr><th colspan="3">11.2 Solving</th></tr>
<tr><td>Graphing</td><td>
Graph each equation separately
Solve for y, then compute some ordered pairs
Use “easy to graph form”
Solve for y, then use calculator for graphing
</td><td>
Solve $\begin{cases} x^2 - 3y = 1 \rightarrow y = -\frac{x^2}{3} - \frac{1}{3} \\ x - y = 1 \rightarrow y = x - 1 \end{cases}$
x | y
0 | -1/3
</td></tr>
<tr><td>Substitution</td><td>
1. Use either equation to solve for 1 variable (pick easiest variable in easiest equation)
2. Substitute expression into the other equation
3. Solve the resulting 1 variable linear equation*
4. Substitute the value(s) form Step 3 into either original equation to find the value of the other variable.
5. Solution is an ordered pair
6. Check all solutions in both original equations
</td><td>
Solve $\begin{cases} x^2 - 3y = 1 \\ x - y = 1 \end{cases}$
1. $y = x - 1$
2. $x^2 - 3(x-1) = 1$
$x^2 + -3x + 2 = 0$
$(x-2)(x-1) = 0$
3. $x = 2, 1$
4. $(2) - y = 1$
$y = 1$
5. $(2,1)$
4. $(1) - y = 1$
$y = 0$
5. $(1,0)$
</td></tr>
<tr><td>Addition</td><td>
1. Rewrite each equation in standard form $Ax + By = C$
2. You want to be able to add the equations and have one variable cancel out. It is usually necessary to multiply one or both equations by a “magic number” so that this will happen.
3. Add equations
4. Find the value of one variable by solving the resulting equation*
5. Substitute the value(s) form Step 4 into either original equation to find the value of the other variable.
6. Solution is an ordered pair
7. Check all solutions in both original equations
</td><td>
Solve $\begin{cases} x^2 + 2y^2 = 10 \\ x^2 - y^2 = 1 \end{cases}$
1. $(They're\ already\ in\ standard\ form)$
2 $-x^2 + -2y^2 = -10\ (multiply\ by\ -1)$
$x^2 - y^2 = 1$
3. $-3y^2 = -9$
$y^2 = 3$
4. $y = \pm\sqrt{3}$
5. $x^2 - (\sqrt{3})^2 = 1$
$x^2 = 1 + 3$
$x = \pm 2$
6. $(\sqrt{3}, 2), (\sqrt{3}, -2)$
5. $x^2 - (-\sqrt{3})^2 = 1$
$x^2 = 1 + 3$
$x = \pm 2$
6. $(-\sqrt{3}, 2), (-\sqrt{3}, -2)$
</td></tr>
</table>

**If the discriminant is negative, then the equations do not intersect*
**If all variables cancel & the resulting equation is true, then the equations are identical*
**If all variables cancel & the resulting equation is false, then the equations do not intersect*

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12 Word Problems

12.1 Proportions, Unknown Numbers, Distance

Proportions

Formula: $\frac{a}{b} = \frac{c}{d}$ Read "a is to b as c is to d"

Example: 3 boxes of CD-Rs cost \$37.47. How much should 5 boxes cost?

Equation: Let x = cost of 5 boxes

$$\frac{3\text{ boxes}}{5\text{ boxes}} = \frac{\text{price of 3 boxes}}{\text{price of 5 boxes}} \quad \text{OR} \quad \frac{3\text{ boxes}}{\text{price of 3 boxes}} = \frac{5\text{ boxes}}{\text{price of 5 boxes}}$$

$$\frac{3\text{ boxes}}{5\text{ boxes}} = \frac{\$37.47}{x} \quad \text{(Use cross product)}$$

$3x = (5)(\$37.47)$

Unknown Numbers

Example: The 1st number is 4 less than the 2nd number. Four times the 1st number is 6 more than twice the 2nd. Find the numbers.

Equation: Let x = 1st number; y = 2nd number

Eq_1: $x = y - 4$

Eq_2: $4x = 6 + 2y$

Distance

Formula: $d = rt$, where d = distance, r = rate, t = time (if rate is mile/hour, then time is hours)

Note: Add or subtract speed of wind or water current with the rate: ($r \pm$ wind) or ($r \pm$ current)

Example: A car travels 180 miles in the same time that a truck travels 120 miles. If the car's speed is 20 mph faster than the trucks speed, find the car's speed and the truck's speed.

Equation: Let x = rate of car; y = rate of truck

	Rate (mph)	Time (hours)	Distance (miles)
Car	x	$180/x$	180
Truck	y	$120/y$	120

Eq_1: $x = 20 + y$

Since $\text{time}_{\text{car}} = \frac{\text{distance}_{\text{car}}}{\text{rate}_{\text{car}}}$; $\text{time}_{\text{truck}} = \frac{\text{distance}_{\text{truck}}}{\text{rate}_{\text{truck}}}$; $\text{time}_{\text{car}} = \text{time}_{\text{truck}}$

Eq_2: $\frac{180}{x} = \frac{120}{y}$

Example: During the first part of a trip, a canoeist travels 48 miles at a certain speed. The canoeist travels 19 miles on the second part of the trip at a speed 5 mph slower. The total time for the trip is 3 hrs. What was the speed on each part of the trip?

Equation: Let x = rate on first part of trip; y = rate on second part of trip

Eq_1: $x = 5 + y$

Since $\text{time}_{\text{total}} = \text{time}_{\text{first part}} + \text{time}_{\text{second part}}$

Eq_2: $3 = \frac{48}{x} + \frac{19}{y+5}$

Remember: you need as many equations as you have variables; attach units to answer if appropriate

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12.2 Money	
Money, Coins, Bills, Purchases	**Formula:** $V_1C_1 + V_2C_2 = V_{total}$, where V = currency value, C = number of coins, bills, or purchased items **Example:** Jack bought black pens at \$1.25 each and blue pens at \$.90 each. He bought 5 more blue pens than black pens and spent \$36.75. How many of each pen did he buy? **Equation:** Let x = number of black pens y = number of blue pens $x + 5 = y$ $\$1.25x + \$.90y = \$36.75$
Break Even Point	**Formula:** $R(x) = C(x)$, where $R(x)$ = total revenue, $C(x)$ = total cost **Example:** A company purchased \$3000 worth of new equipment so that it could produce wigits. The cost of producing a wigit is \$3.00, and it is sold for \$5.50. Find the number of wigits that must be sold for the company to break even. **Equation:** Let x = number of wigits $C(x)$ = total cost for producing x wigits = $\$3000 + \$3.00x$ $R(x)$ = total revenue for selling x wigits = $\$5.50x$ $\$5.50x = \$3000 + \$3.00x$
Simple Interest	**Formula:** $A = P + Prt$, where A = total amount of money when using simple interest, P = principal, r = rate of interest, t = duration
Compound Interest	**Formula:** $A = P\left(1+\frac{r}{n}\right)^{nt}$, where A = total amount of money when interest is compounded, P = principal, r = annual rate of interest, n = number of times interest is compounded each year, t = duration in years **Example:** How much money will there be in an account at the end of 10 years if \$14,000 is deposited at 7%? The interest is compounded quarterly. **Equation:** $A = \$14,000\left(1+\frac{.07}{4}\right)^{(4\cdot 10)}$ **Example:** How much long will it take \$1,000 to grow to \$10,000 if the interest rate is 15% and it is compounded quarterly? **Equation** $\$10,000 = \$1,000\left(1+\frac{.15}{4}\right)^{(4\cdot t)}$ $10 = \left(1+\frac{.15}{4}\right)^{(4\cdot t)}$ $\log_{\left(1+\frac{.15}{4}\right)} 10 = 4t$
Continuously Compounded Interest	**Formula:** $A = Pe^{rt}$, where A = total amount of money when interest is continuously compounded, r is the annual interest rate for P dollars invested for t years, e = 2.71828... (use the calculator button e)

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12.3 Sum of Parts

Work

Formula: $R_1 + R_2 = R_T$**,** where R_1 is the rate/hour of one person; R_2 is the rate/hour of the 2nd person, R_T is their rate/hour when they work together. The total time it takes them to complete the job together is $1/R_T$

Example: Together 2 painters paint the room in 6 hours. Alone, the experienced painter can paint the room 2 hours faster than the newbie. Find the time which each person can paint the room alone.

Equation: Let x = total time of experienced painter to complete job
y = total time of newbie painter to complete job

	Total Time to Complete Job (hours)	**Part of Job Completed in 1 Hour (rate/hour)**
Experienced Painter	x	$1/x$
Newbie	y	$1/y$
Together	6	1/6

$$y - 2 = x$$

$$\frac{1}{x} + \frac{1}{y} = \frac{1}{6}$$

Mixture

Formula: $V_1P_1 + V_2P_2 = V_FP_F$, where V_1 = 1st volume, P_1 = 1st percent solution, V_2 = 2nd volume, P_2 = 2nd percent solution, V_F = final volume, P_F = final percent solution

Example: A pharmacist needs 70 liters of a 50% alcohol solution. She has available a 30% alcohol solution and an 80% alcohol solution. How many liters of each solution should she mix to obtain 70 liters of a 50% alcohol solution?

Equation: Let x = amount of 30% solution; y = amount of 80% solution

	Amount of alcohol solution (liters)	**Alcohol Strength**	**Amount of pure Alcohol (liters)**
30% Solution	x	30%	$.30x$
80% Solution	y	80%	$.80y$
50% Solution	70	50%	(.50)(70) = 35

$$x + y = 70$$

$$.30x + .80y = 35$$

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13 Calculator

13.1 Buttons	
Calculator buttons	▪ ^ – "Carrot", to raise a number to a power. EX: -27 ^ 1/3 ≠ -27 ^ (1/3) ▪ MATH →Contains some power and root computations ▪ (-) – To make a number negative. EX: (-)6 ▪ 2nd [QUIT] – ***"When in doubt, QUIT and go HOME!"***
Calculator not working?	▪ Run defaults to reset calculator. 2nd [MEM] **7 → 1 → 2**
Storing & recalling information	▪ 2nd (-) [ANS] – Displays the last answer The calculator will automatically put in [ANS] when the following keys are used at the beginning of a new line: + – × ÷ ^ ▪ 2nd [ENTRY] – Displays the previous user entry ▪ 7 STO ALPHA MATH [A] – Stores the number 7 in the variable A ▪ 2nd [RCL] ALPHA MATH [A] – Displays the number 7

13.2 Rounding

1. When a number is written in scientific notation, the number of digits used is the number of significant digits.
2. Performing a calculation does not give more accuracy then the numbers used to make the calculator; the results should not suggest that it does
   - When performing , ÷, $\sqrt{\ }$, ^: the answer contains the minimum number of significant digits of the numbers used
   - When performing +, -: the answer is accurate to the same place as the least accurate number used in the calculation
3. Do not round until the end of a problem. When using a calculator, use the stored values in the calculator (the calculator carries more digits than it sees)

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13.3 The Window	
Setting the exact window format WINDOW	1. Press WINDOW 2. Change the appropriate setting WINDOW Xmin = pick min *x* value Ymin = pick min *y* value Xmax = pick max *x* value Ymax = pick max *y* value Xscl = pick *x* axis increment Yscl = pick *y* axis increment
Adjusting the viewing window ZOOM (you can press WINDOW at any time to see what range you are looking at)	▪ **ZBox** – zooms on the box you specify. 1. Press ZOOM **1** 2. Move curser to one corner of zoom box and press ENTER 3. Move curser to the opposite corner of zoom box and press ENTER ▪ **Zoom In** - magnifies the graph around the cursor location 4. Press ZOOM **2** 5. Use arrow keys to position the cursor on the portion of the line that you want to zoom in on 6. To zoom in, press ENTER 7. You can continue to zoom by just pressing ENTER – or - by moving the cursor and pressing ENTER ▪ **Zoom Out** – does the reverse of zoom in 1. Press ZOOM **3** 2. Follow the preceding instructions ▪ **ZStandard** – will change your viewing screen so that both x and y will go from -10 to 10. 1. Press ZOOM **6** ▪ **ZSquare** – will change your viewing window so that the spacing on the tick marks on the x-axis is the same as that on the y-axis. A circle will display properly, not as an oval. 1. Press ZOOM **5** ▪ **ZDecimal –** lets you trace a curve by using the numbers 0, .1, .2, … for x. Zdecimal will change your viewing screen so that x will go form -4.7 to 4.7. 1. Press ZOOM **4** ▪ **ZInteger –** lets you trace a curve by using the numbers 0, 1, 2,… for x. ZInteger can be used for any viewing window,. 1. Press ZOOM **8** ENTER ▪ **ZoomStat** – will change your viewing window so that you can see a scattergram of points that you have entered in the statistic editor. 1. Press ZOOM **9** ▪ **ZoomFit** – will adjust the dimensions of the y-axis to display as much of a curve as possible. The dimensions of the x-axis will remain unchanged. A good option to use if you can't see the graph. 1. Press ZOOM **0**

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<table>
<tr><th colspan="2">13.4 Graphing, Finding Function Values</th></tr>
<tr><td>Graphing functions</td><td>1. Put equation in "Y equals" form
2. Y = → Y_1= enter function (Use X,T,O,N to enter the variable x)
3. GRAPH – you might have to change the viewing window, see previous table</td></tr>
<tr><td>Turning an Equation On or Off</td><td>1. Y =
2. Move the cursor to the equation whose status you want to change.
3. Use to place the cursor over the "=" sign of the equation.
4. Press ENTER to change the status.</td></tr>
<tr><td>Finding function values with TRACE</td><td>▪ To display points along the graph one at a time… (The curser is forced to remain exactly on the graph. The position of the curser will be displayed as an ordered pair in the viewing window.)
1. TRACE
2. & Increase and decrease the x-values.
& Switch between multiple graphs.</td></tr>
<tr><td>Finding function values with 2nd [CALC]</td><td>▪ To determine the y-value, given an x-value…
1. 2nd [CALC] → 1: value –enter any x (if you enter 0 for x, the y-intercept will be display)
2. If the x or y value is not displayed in "graph mode", an error message is returned. Adjust your viewing window.
▪ To determine the x-intercept (or root)…
1. 2nd [CALC] → 2: zero
2. Move curser to the left of the x-intersect and hit ENTER
3. Move curser to the right of the x-intersect and hit ENTER
4. Hit ENTER for guess, then the x-intercept will be displayed
▪ To determine the minimum/maximum (the vertex of a parabola)…
1. 2nd [CALC] → 3: minimum/4: maximum
2. Move curser to the left of the min/max and hit ENTER
3. Move curser to the right of the min/max hit ENTER
4. Hit ENTER for guess, then the local minimum/maximum will be displayed</td></tr>
<tr><td>Finding function values with 2nd [TABLE]</td><td>▪ To show graph(s) in table format…
1. 2nd [TBL SET] to display the TABLE SETUP menu

TABLE SETUP
TblStart = pick start value
Δ Tbl = pick increment to increase start value by
Indpnt: Auto Ask
Depend: Auto Ask

2. 2nd [TABLE]</td></tr>
</table>

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13.5 Scattergrams and Linear Regression	
Scatter-grams and Performing Linear Regression	1. **Clear L1 and L2**: STAT **1** → Move to L1 with arrow keys → CLEAR ENTER → Move to L2 with arrow keys → CLEAR ENTER 2. **Enter data into L1** (independent variable) **and L2** (dependent variable): use arrow keys 3. **Turn on STAT PLOT**: Y = → with arrow keys move to PLOT1 → ENTER 4. **Clear any graphs**: Y = → with arrow keys move to function lines → CLEAR 5. **Show linear correlation coefficient *r***: 2nd [CATALOG] → Scroll down to *DiagnosticOn* → ENTER ENTER 6. **Find regression line:** STAT → CALC **4** → VARS → Y-VARS **1** ENTER 7. **Graph**: ZOOM **9** 8. **Move along the points or line**: TRACE
Customizing the Plotter	1. 2nd [STAT PLOT]

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13.6 Expressions, Equations and Inverses	
Testing for equivalent expressions	1. Y = to display the EQUATION menu Plot1 Plot2 Plot3 \ Y_1 = enter 1st expression θY_2 = enter 2nd expression *(move the curser over "\" and hit* ENTER *4 times to change it to "θ")* 2. GRAPH *(The graphs will be identical if they are equal)*
Solving a 1 variable equation	1. Y = to display the EQUATION menu Plot1 Plot2 Plot3 \ Y_1 = enter right side of equation \ Y_2 = enter left side of equation 2. GRAPH *(The x values where the graphs intersect are the solutions)*
Solving systems of equations	1. Y = to display the EQUATION menu Plot1 Plot2 Plot3 \ Y_1 = enter 1st function \ Y_2 = enter 2nd function 2. Use the intersect option to find where the two graphs intersect ▪ 2nd [CALC] → *5: intersect* ▪ Move spider close to the intersection & hit ENTER 3 times ▪ An ordered pair will be displayed
Determining if 2 functions are inverses of each other	1. Y = to display the EQUATION menu Plot1 Plot2 Plot3 \ Y_1 = enter 1st function \ Y_2 = enter 2nd function \ Y_3 = X,T,Θ,N 2. ZOOM → *5: ZSquare* 3. GRAPH *(If Y_1 and Y_2 are symmetric about Y_3, then they are inverses of each other)*

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14 Big Picture

14.1 Topic Overview

Equations

- Linear: $y = x + 1$
- Quadratic: $x^2 + 2x - 1 = 0$
- Higher Degree: $x^4 = 16$
- Radical: $\sqrt{x} = 4$
- Rational: $1 = \frac{5}{x}$
- Exponential: $2^x = 8$
- Logarithmic: $\log_2 8 = x$

Inequalities

- Compound: $-2 \le x \le 0$
- Linear (1 variable): $x \le 0$
- Linear (2 variable): $y < x + 1$
- Systems of Inequalities: $\begin{cases} y \le x \\ y \ge x - 2 \end{cases}$

Additional Topics

- Sets: 0 $[0, \infty)$ $\{x \mid x \ge 0\}$
- Functions: $f(x) = 2x$
- Inverse Functions: $f^{-1}(x) = \frac{x}{2}$
- Systems of Linear Equations: $\begin{cases} y = x \\ y = x + 1 \end{cases}$
- Systems of Non-Linear Equations: $\begin{cases} x^2 + y^2 = 1 \\ y = x + 1 \end{cases}$
- Word Problems
- Calculator Use

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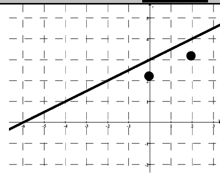
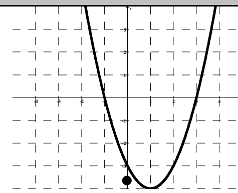
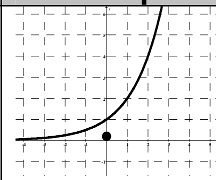
14.2 Linear vs. Quadratic vs. Exponential			
	Linear	**Quadratic**	**Exponential**
Graph – function with infinite solutions	 Line	 Parabola •	 Increasing or decreasing at a faster and faster rate
General form	$y=\frac{1}{2}x+3$ Slope (m) y-intercept	a b c $y=1x^2-2x-3$ $y=(x-1)^2-4$ h k	$y=(1)(2)^x$ y-intercept base multiplier
Graphing with critical information	1. Plot y-intercept 2. Start at y-intercept, move vertically the numerator of the slope, and horizontally the denominator of the slope	1. Plot the vertex $\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$ or (h,k) 2. Plot an additional point 3. Draw symmetric parabola	1. Plot the y-intercept 2. Plot points on both sides
Graphing with x-y chart – get equation in "*y* *equals form*"	Any three points will do **x** / **y** -1 / 2.5 0 / 3 1 / 3.5	Pick the vertex and points around it ***x*** / ***y*** 0 / -3 1 / -4 2 / -3	***x*** / ***y*** -2 / 1/4 -1 / 1/2 0 / 1 1 / 2 2 / 4
Domain	All real numbers	All real numbers	All real numbers
Range	All real numbers	$y \geq -4$	$y > 0$
***y*-intercept (0,*y*)**	If the equation is in "*y* equals" form, it is easy to set *x*=0 and solve		
***x*-intercept (*x*,0)**	See "Solving for *x*, given *y*" below. Set *y=0*.		
Solving for *y*,	Substitute in the value for *x* and compute		
Solving for *x*,	• Get *x* by itself with "undoing" operations	• Use square root property • Get equation equal to zero, then factor or use the quadratic formula	• Get like bases and set exponents equal • "log" both sides and solve for *x*
Determining the equation	• Find the slope and the *y*-intercept	• Substitute the vertex in (*h*,*k*) form	• Find the *y*-intercept and the base multiplier
Examples	• Find equation of line with a slope of *m* going thru (*x*,*y*)	• The path of a ball as a function of time	• Compound interest • Exponential decay
Inverse – swap *x* & *y*, solve for *y*.	$x=\frac{1}{2}y+3$ $f^{-1}(x)=2x-6$	No inverse, not one-to-one	$x=(1)2^y$ $f^{-1}(x)=\log x/\log 2$

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14.3 Undo Any 1 Variable Equation

	Linear	Quadratic	Rational Exponent	Exponential	Logarithmic
Example	$32 = (9/5)C + 32$	$200 = 100(1+r)^2$	$2x^{3/2} = 16$	$2^x = 5$	$\log(x) = 5$
Simplify					
Addition/ Subtraction	$0 = (9/5)C$				
Multiplication/ Division	$(5/9)0=(9/5)C(5/9)$ $C = 0$	$200/100 = (1+r)^2$	$x^{3/2} = 8$		
Power/Root		$\text{sqrt}(2) = (1+r)$	$(x^{3/2})\text{^}(2/3)=8\text{^}(2/3)$ $x = 4$		
Exponential/ Logarithms				$\log_2 2^x = \log_2 5$ $x = \log_2 5$	$10^{\log(x)} = 10^5$ $x = 100,000$
Parenthesis		$r = \text{sqrt}(2) - 1$			

order of operations

undo

- If both sides of an equation are raised to the same power, **all** solutions of the original equation are **among** the solutions of the new equations.
- If a root greater than 2 is taken of both sides of an equation, it is possible not all solutions will be obtained

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